Special Binomial Products

- **Special binomial products – squaring binominals**

Special Products	Formula	Initial Expansion	Example
difference of squares	$(a+b)(a-b)=a^2-b^2$ It does not matter if $(a-b)$ comes first	$(a+b)(a-b)=a^2-ab+ba-b^2$ $=a^2-b^2$	$(x+2)(x-2)=x^2-2^2=x^2-4$ or $(x-2)(x+2)=x^2-2^2=x^2-4$ $(a=x,\ b=2)$
square of sum	$(a+b)^2 = a^2+2ab+b^2$ A perfect square trinomial	$(a+b)^2=(a+b)(a+b)$ $=a^2+ab+ba+b^2$ $=a^2+2ab+b^2$	$(x+3)^2=x^2+2\cdot x\cdot 3+3^2$ $=x^2+6x+9$
square of difference	$(a-b)^2 = a^2-2ab+b^2$ A perfect square trinomial	$(a-b)^2=(a-b)(a-b)$ $=a^2-ab-ba+b^2$ $=a^2-2ab+b^2$	$(x-4)^2=x^2-2\cdot x\cdot 4+4^2$ $=x^2-8x+16$

- **Special binomial products:** special forms of binomial products that are worth memorizing.
- **Memory aid:** $(a\pm b)^2=(a^2\pm 2ab+b^2)$

Example: Find the following products.

1. $(3y+4)(3y-4)=(\overset{\overset{a}{\downarrow}}{3y})^2-\overset{\overset{b}{\downarrow}}{4}^2$

   $=9y^2-16$

   $(a+b)(a-b)=a^2-b^2$

   $a=3y\ ,\quad b=4$

2. $\left(5t+\frac{1}{2}\right)^2=(5t)^2+2(5t)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2$

   $=25t^2+5t+\frac{1}{4}$

   $(a+b)^2=a^2+2ab+b^2$

   $a=5t\ ,\quad b=\frac{1}{2}$

3. $(3q-\frac{1}{6}p)^2=(3q)^2-2(3q)\left(\frac{1}{6}p\right)+\left(\frac{1}{6}p\right)^2$

   $=9q^2-q\,p+\frac{1}{36}p^2$

   $(a-b)^2=a^2-2ab+b^2$

   $a=3q\ ,\quad b=\frac{1}{6}p$

4. $(t+1)^3=(t+1)^2(t+1)$

   $=(t^2+2t+1)(t+1)$

   $=t^3+t^2+2t^2+2t+t+1$

   $=t^3+3t^2+3t+1$

   $a^na^m=a^{n+m}$

   $(a+b)^2=a^2+2ab+b^2$

   Distribute

   Combine like terms.

5. $(\underset{\overset{\uparrow}{a}}{2A-3}+\underset{\overset{\uparrow}{b}}{4B})(\underset{\overset{\uparrow}{a}}{2A-3}-\underset{\overset{\uparrow}{b}}{4B})=(2A-3)^2-(4B)^2$

   $=(2A)^2-2(2A)\cdot 3+3^2-16B^2$

   $=4A^2-12A+9-16B^2$

   $(a+b)(a-b)=a^2-b^2:\ a=2A-3,\ b=4B$

   $(a-b)^2=a^2-2ab+b^2:\ a=2A,\ b=3$

   Simplify

- **Using function notation:**

Example: Given $f(x)=-3x+x^2$, find and simplify **1.** $f(u-1)$, and **2.** $f(a+h)-f(a)$.

1. $f(u-1)=-3(u-1)+(u-1)^2$

   $=-3u+1+u^2-2u+1$

   $=u^2-5u+2$

   Replace x with $(u-1)$

   $(a-b)^2=a^2-2ab+b^2$

   Combine like terms.

2. $f(a+h)-f(a)=[-3(a+h)+(a+h)^2]-(-3a+a^2)$

   $=\cancel{-3a}-3h+\cancel{a^2}+2ah+h^2+\cancel{3a}-\cancel{a^2}$

   $=h^2+2ah-3h$

   Replace x with $(a+h)$ and a.

   Remove parentheses.

   Combine like terms.