# Algebra Test Prep and Review

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Evaluating Polynomial Functions

- **Polynomial function:** The expression used to describe the function is a polynomial.

  **Example:** \( f(x) = 2x^3 - 3x^2 + 7x + 8 \)
  \( g(x) = -3x^4 + 5x^2 - 2 \)

  \[ f(x) \text{ & } g(x) \text{ are functions.} \]

- **Evaluating polynomial functions**

  **Example:** 1. If \( f(x) = 2x^3 + 1 \), find \( f(2) \) and \( f(-1) \).

     \[ f(2) = 2(2)^3 + 1 = 16 + 1 = 17 \]
     \[ f(-1) = 2(-1)^3 + 1 = -2 + 1 = -1 \]

  2. If \( R(x) = -8x^3 + x^2 + 2 \), find \( R(0) \) and \( R\left(\frac{1}{2}\right) \).

     \[ R(0) = -8(0)^3 + (0)^2 + 2 = 2 \]
     \[ R\left(\frac{1}{2}\right) = -8\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + 2 \]
     \[ = -1 + \frac{1}{4} + 2 = \frac{5}{4} \]

  **Example:** The polynomial function \( C(x) = 3,000 + 0.5x^2 \) can be used to determine the total cost (in dollars) of producing \( x \) laptops in an electronics firm.

  1. What is the total cost of producing 10 laptops?
  2. Use the following graph to estimate \( C(40) \).

  **Solution:** 1. \( C(10) = 3,000 + 0.5(10)^2 \)

     \[ = 3,000 + 50 \]
     \[ = 3,050 \]

     \( C(x) = 3,000 + 0.5x^2 \), replace \( x \) with 10.

     \( x = \text{Number of laptops} \)

  2. \( C(40) \): locate \( x = 40 \) on the \( x \) axis, move vertically to the graph, and then move horizontally to the \( C(x) \) axis. Thus \( C(40) \approx 3,800 \).
Adding and Subtracting Polynomials

**Adding or subtracting polynomials**

*Example:* Find the sum of \(2x^3 - 3x^2 + x - 4\) and \(x^3 + 4x^2 + 2x + 1\).

**Steps**

- Regroup like terms.
- Combine like terms.

**Solution**

\[(2x^3 - 3x^2 + x - 4) + (x^3 + 4x^2 + 2x + 1)\]

\[= (2x^3 + x^3) + (-3x^2 + 4x^2) + (x + 2x) + (-4 + 1)\]

\[= 3x^3 + x^2 + 3x - 3\]

*Example:* Find the difference of \(5x^2 + 4x - 2\) and \(2x^2 - 3x + 13\).

**Steps**

- Remove parentheses. (Reverse each sign in second parentheses.)
- Regroup like terms.
- Combine like terms.

**Solution**

\[(5x^2 + 4x - 2) - (2x^2 - 3x + 13)\]

\[= 5x^2 + 4x - 2 - 2x^2 + 3x - 13\]

\[= (5x^2 - 2x^2) + (4x + 3x) + (-2 - 13)\]

\[= 3x^2 + 7x - 15\]

**Column method**

*Example:* Find the sum of \(3x^3 - 5x^2 + 7x - 3\) and \(2x^3 + 3x + 5\).

**Steps**

- Line up like terms in columns.
- Add.

**Solution**

\[3x^3 - 5x^2 + 7x - 3 + 2x^3 + 3x + 5\]

\[= 5x^3 - 5x^2 + 10x + 2\]

*Example:* Find the difference of \((5x^2 - 2x + 3) - (2x^2 - 5)\).

**Steps**

- Line up like terms in columns:
- Change signs in minuend and add.

**Solution**

\[5x^2 - 2x + 3 \quad \text{Subtrahend}\]

\[-2x^2 + 5 \quad \text{Minuend}\]

\[3x^2 - 2x + 8 \quad \text{Difference}\]

**The opposite of the polynomial:**

\(-p\): the opposite of the polynomial \(p\): polynomial \(p + (-p) = 0\)

*Example:* Write two expressions for the opposite of the polynomial \(7a^4b^2 - 3a^3b - 4a^2\)

**Solution:** opposite expression: \(- (7a^4b^2 - 3a^3b - 4a^2)\)

or \(-7a^4b^2 + 3a^3b + 4a^2\)

Replace each term with its opposite.
Factoring Polynomials by Grouping

Steps for factoring by grouping:

- Group terms with the GCF.
- Factor out the GCF from each group.
- Factor out the GCF again from the last step.

Factoring completely: Continue factoring until no further factors can be found.

Example: Factor the following completely.

1. \(6ab^2 - 3a^2b + 2b - a = (6ab^2 - 3a^2b) + (2b - a)\)
   \(= 3ab(2b - a) + (2b - a) \cdot 1\)
   \(= (2b - a)(3ab + 1)\)

2. \(2ab + bc - 2bc + 4ab = (2ab + 4ab) + (bc - 2bc)\)
   \(= 6ab - bc\)
   \(= b(6a - c)\)

3. \(x^3 - xy^2 - x^2y + y^3 = (x^3 - x^2y) - (xy^2 - y^3)\)
   \(= x^2(x - y) - y^2(x - y)\)
   \(= (x - y)(x^2 - y^2)\)
   \(= (x - y)(x + y)(x - y)\)
   \(= (x - y)^2(x + y)\)

Tip: Recognize factoring patterns, such as \(2b - a\), \(x - y\), ...

4. \(32x^3y - 2xy^3 = 2xy(16x^2 - y^2)\)
   \(= 2xy[(4x)^2 - y^2]\)
   \(= 2xy(4x + y)(4x - y)\)