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Unit 2 Summary

- **Equation:** a mathematical statement that contains two expressions separated by an equal sign.
- **Solution, root or zero of an equation:** a solution is the particular value of the variable in the equation that makes the equation true.
- **Solution Set { }:** the set of all values that makes the equation true.
- **Linear equation (or first-degree equation) in one variable:** an equation in which the highest **power** of the **variable** is **one**. (An equation whose graph is a straight line.)
- **Equations of different degrees**

Equation	Standard Form	Example	Comments
first-degree equation (linear equation)	$Ax + B = 0$ $(x = x^1)$	$5x + 4 = 0$	The highest power of x is 1.
second-degree equation (quadratic equation)	$Ax^2 + Bx + C = 0$	$2x^2 + 7x - 3 = 0$	The highest power of x is 2.
third-degree equation (cubic equation)	$Ax^3 + Bx^2 + Cx + D = 0$	$3x^3 + 4x^2 - 8x + 1 = 0$	The highest power of x is 3.
fourth-degree equation	$Ax^4 + Bx^3 + Cx^2 + Dx + E = 0$	$x^4 - 9x^3 + 3x^2 + 2x - 5 = 0$	The highest power of x is 4.

- **Higher-degree equations are nonlinear equations.**
- **A linear equation in two variables:** an equation that contains two variables in which the highest power (exponent) of two **variables** is one
- **Formula:** an equation that contains more than one variable and is used to solve practical problems in everyday life.
- **An inequality:** a mathematical statement that contains $<$, $>$, $\geq$, or $\leq$ symbol.
- **Solution of an inequality:** the particular value(s) of the variable in the inequality that makes the inequality true.
- **Compound inequality:** a statement that contains more than one inequality. $a < x < b$
- **Absolute value equation:** an equation that includes absolute value(s).
- **$|x| = A$ is equivalent to $x = \pm A$.** **Example**: $|5t - 3| = 2$ is equivalent to $5t - 3 = \pm 2$.

- **Equations involving decimals**
  - Multiply each term by a power of 10 (10, 100, 1,000, etc.) to clear the decimals.
  - Collect the variable terms on one side of the equation and the constants on the other side.
  - Isolate the variable.
- **Equations involving fractions**
  - Multiply each term by the LCD.
  - Collect the variable terms on one side of the equation and the constants on the other side.
  - Isolate the variable.
- **Properties for solving equations**

Properties	Equality	Example
property of addition	$A = B,\ A + C = B + C$	Solve $y - 7 = 2$ $y - 7 + 7 = 2 + 7,\ y = 9$
property of subtraction	$A = B,\ A - C = B - C$	Solve $x + 3 = -8$ $x + 3 - 3 = -8 - 3,\ x = -11$
property of multiplication	$A = B,\ A \cdot C = B \cdot C$ $(C \neq 0)$	Solve $\frac{-t}{6} = 7$ $\frac{-t}{6}(-6) = 7(-6),\ t = -42$
property of division	$A = B,\ \frac{A}{C} = \frac{B}{C}$ $(C \neq 0)$	Solve $4a = -16$ $\frac{4a}{4} = \frac{-16}{4},\ a = -4$

- **Equation-solving strategy**

Equation-Solving Strategy

- Clear the fractions or decimals if necessary.
- Remove parentheses.
- Combine like terms on each side of the equation if necessary.
- Collect the variable terms on one side of the equation and the numerical terms on the other side.
- Isolate the variable.
- Check the solution with the original equation.

- **Steps for solving word problems**

Steps for Solving Word Problems

- Organize the ***facts*** given from the problem.
- Identify and label the unknown quantity (***let x = unknown***).
- Draw a ***diagram*** if it will make the problem clearer.
- Convert words into a mathematical ***equation***.
- ***Solve*** the equation and find the solution(s).
- ***Check*** and state the ***answer***.

- **Sets Summary**

Unions, Intersections, and Subsets		Example
union of A and B ($A \cup B$) OR	The set of all elements contained in A or B, or both.	If $A = \{2, 5\}$ and $B = \{1, 3, 4\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$.
intersection of A and B ($A \cap B$) AND	The set of all elements contained in both A and B.	If $A = \{3, 6, 9\}$ and $B = \{5, 6, 7, 8, 9\}$ then $A \cap B = \{6, 9\}$.
empty set (or null set) $\emptyset$	A set that contains no elements.	If $A = \{ x \mid x = \text{Feb. } 30\}$, then $A = \emptyset$.
subset ($B \subset A$)	The subset B is a portion of another set A.	If $A = \{2, 5, 7, 11\}$, $B = \{5, 11\}$, then ($B \subset A$).
$x \in A$	x is an element of the set A.	$\frac{2}{3} \in$ Rational numbers
$x \notin A$	x is not an element of the set A.	$\sqrt{5} \notin$ Rational numbers

- **Properties of absolute value**

Absolute Value		Example
absolute value	$\lvert x\rvert = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$	If $\lvert 2x - 3\rvert = 5$ Then $2x - 3 = 5$ or $2x - 3 = -5$
properties	$\lvert xy\rvert = \lvert x\rvert\lvert y\rvert$	$\lvert -4a\rvert = \lvert -4\rvert\lvert a\rvert = 4\lvert a\rvert$
	$\left\lvert \frac{x}{y} \right\rvert = \frac{\lvert x\rvert}{\lvert y\rvert}$ $(y \neq 0)$	$\left\lvert \frac{3x^3}{5y} \right\rvert = \frac{\lvert 3x^3\rvert}{\lvert 5y\rvert} = \frac{3\lvert x^3\rvert}{5\lvert y\rvert}$

- **Procedure to solve an absolute value equation:**
  - Isolate the absolute value.
  - Remove the absolute value symbol and set up two equations (one positive and one negative.)
  - Solve two equations.
  - Check.

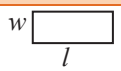
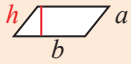
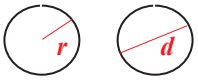
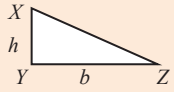
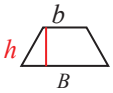
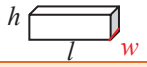
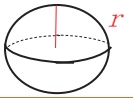
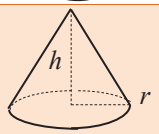
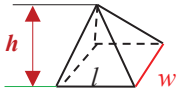
- **Absolute value inequalities summary**

Absolute Value Inequality		Example
$\lvert x\rvert < A$	$-A < x < A$ or $(-A, A)$	$\lvert x\rvert < 2$: $\{ x \mid -2 < x < 2\}$ or $(-2, 2)$
$\lvert x\rvert \leq A$	$-A \leq x \leq A$ or $[-A, A]$	$\lvert x\rvert \leq 2$: $\{x \mid -2 \leq x \leq 2 \}$ or $[-2, 2]$
$\lvert x\rvert > A$	$x < -A$ or $x > A$ or $(-\infty, -A) \cup (A, \infty)$	$\lvert x\rvert > 2$: $\{x \mid x < -2$ or $x > 2\}$ or $(-\infty, -2) \cup (2, \infty)$
$\lvert x\rvert \geq A$	$x \leq -A$ or $x \geq A$ or $(-\infty, -A] \cup [A, \infty)$	$\lvert x\rvert \geq 2$: $\{x \mid x \leq -2$ or $x \geq 2\}$ or $(-\infty, -2] \cup [2, \infty)$

- **Business formulas**

Application	Formula
Percent Increase	Percent increase $= \frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{\text{N} - \text{O}}{\text{O}}$
Percent Decrease	Percent decrease $= \frac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \frac{\text{O} - \text{N}}{\text{O}}$
Sales Tax	sales tax = sales $\times$ tax rate
Commission	commission = sales $\times$ commission rate
Discount	discount = original price $\times$ discount rate sale price = original price – discount
Markup	markup = original price $\times$ markup rate original price = selling price – markup
Simple Interest	interest = principle $\times$ interest rate $\times$ time, $I = P\,r\,t$ balance = principle + interest
Compound Interest	balance = principle $(100\% + \text{interest rate})^t$ balance $= P(100\% + r)^t$

- **Recall some geometry formulas**

Name of the Figure	Formula	Figure
rectangle	$P = 2l + 2w$ $A = lw$	
parallelogram	$P = 2a + 2b$ $A = bh$	
circle	$C = \pi d = 2\pi r$ $A = \pi r^2$	
triangle	$\angle X + \angle Y + \angle Z = 180^0$ $A = \frac{1}{2}bh$	
trapezoid	$A = \frac{1}{2}h(b + B)$	
cube	$V = s^3$	
rectangular solid	$V = lwh$	
cylinder	$V = \pi r^2 h$	
sphere	$V = \frac{4}{3}\pi r^3$	
cone	$V = \frac{1}{3}\pi r^2 h$	
pyramid	$V = \frac{1}{3}lwh$	

- **More formulas**

Application	Formula	Component
distance	$d = rt,\ r = \frac{d}{t},\ t = \frac{d}{r}$	d – distance r – speed t – time
simple interest	$I = P\,r\,t,\ P = \frac{I}{rt},\ t = \frac{I}{Pr}$	I – interest P – principle r – interest rate (%) t – time (years)
compound interest	$B = P\,(100\% + r)^{\,t}$	B – balance P – principle r – interest rate (%) t – time (years)
percent increase	$\frac{N - O}{O}$	N – new value O – original value
percent decrease	$\frac{O - N}{O}$	N – new value O – original value
sale price	$S = L - r\,L,\ L = \frac{S}{1-r}$	S – sale price L – list price r – discount rate
intelligence quotient (I.Q.)	$I = \frac{100m}{c}$	I – I.Q. m – mental age c – chronological age
temperature	$C = \frac{5}{9}(F - 32),\ F = \frac{9}{5}C + 32$	C – Celsius F – Fahrenheit

PRACTICE QUIZ

Unit 2 Equations and Inequalities

1. Solve the following equations.

a. $3(x - 2) + 4x - 7 = 3(5 - x)$

b. $0.3y - 0.27 = -4.36y$

c. $\frac{3x}{4} - \frac{2}{3} = \frac{x}{2} + \frac{1}{4}$

2. Five less than four times a number is nine more than the number divided by two. Find the number.

3. Find three consecutive even integers such that four times the first integer is two less than the sum of the second and third integers.

4. Two vehicles are 340 km apart and are traveling towards each other. Their speeds differ by 10 km per hour. What is the speed of each vehicle if they meet after 2 hours?

5. Alice boats at a speed of 26 km per hour in still water. The river flows at a speed of 12 km per hour. How long will it take Alice to boat 4 km downstream? 3 km upstream?

6. Tom purchased 46-cent, 66-cent, and 86-cent Canadian stamps with a total value of $6.80. If the number of 66-cent stamps is 3 more than the number of 46-cent stamps, and the number of 86-cent stamps is 2 more than one half the number of 46-cent stamps. How many stamps of each did Tom receive?

7. Solve the following inequalities and graph the solution sets.

a. $-7x - 3 \geq 11$

b. $3 - 2(4x - 5) + 7x > 2x + 10$

c. $\frac{3}{4}(5 - y) - \frac{5}{2} \leq \frac{1}{3}$

8. Amanda got a 78% on the midterm exam in English. To get a B+, the average of her midterm and final exam must be between 76% and 80%. For what range of scores on the final exam will Amanda need to get a B+?

9. Indicate whether each of the following is true or false.

a. $\frac{-5}{16} \in$ rational numbers

b. $\sqrt{13} \in$ rational numbers

10. **a**. Given $A = \{ a \mid a$ is a prime number between 10 and 18$\}$

$B = \{ b \mid b$ is a number between 12 and 16$\}$

List the numbers in $A \cup B$ and $A \cap B$

b. Given $A = \{3, 5, 7\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{-3, -2\}$. List the elements in the following:

$A \cup B$
$A \cap B$
$A \cap C$

11. Solve the following and graph the solution set.

$-3 < \frac{1+2x}{3} \leq 1$

12. Solve the following equations.

a. $2|x + 3| - 4 = 6$

b. $|3x - 4| = |5x - 2|$

13. Solve the following inequalities.

a. $|3x - 4| \leq 7$

b. $|5x - 3| > 4$