

73. ANSWER: 22.36 units

SOLUTION:  $\overline{AD}$  is the hypotenuse of a right triangle, where 5 is one leg and  $\frac{5}{2}$  is the other. This is the case for each segment of ABCD. Therefore, each side length of ABCD will have length  $\sqrt{5^2 + 2.5^2} \approx 5.59$ . Therefore, the perimeter is  $4 \times 5.59$ , or  $\approx 22.36$  units.

3 possible points

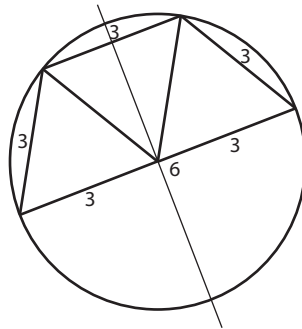
1 point (content): Realize  $\overline{AB}$  is the hypotenuse of a right triangle.

1 point (content): All arithmetic is correct.

1 point (clarity): The explanation is clearly written.

74. ANSWER: 15 units.

SOLUTION: Dividing the trapezoid shows three equilateral triangles each with side lengths of three units. Therefore, the perimeter of the trapezoid is 15 units.



3 possible points

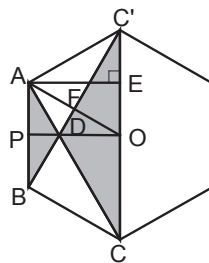
1 point (content): Realize the need to use equilateral triangles.

1 point (content): Arithmetic is correct.

1 point (clarity): The explanation is clearly written.

75. ANSWER:  $24\sqrt{3}$ , or about 41.57 units

SOLUTION:



Assume that each side of the hexagon has length  $x$ . Label the center of the hexagon  $O$ , the midpoint of  $\overline{AB}$  as  $P$ , and the vertex opposite  $C$  as  $C'$ . Since the hexagon is regular, segment  $\overline{C'C}$  is straight, and  $\overline{OP}$  is a mirror segment for polygon  $C'APO$  and polygon  $CBPO$ . Because  $\overline{AD}$  and  $\overline{BD}$  are congruent,  $D$  lies on  $\overline{OP}$  and  $\overline{PDO}$  is the perpendicular bisector of  $\overline{AB}$  and  $\overline{CC'}$ . Construct  $\overline{AE}$  perpendicular to  $\overline{CC'}$ .  $\angle AC'O$  must be  $60^\circ$ , making  $\triangle AC'E$  a 30-60-90 right triangle. Since a regular hexagon can be made up of six equilateral triangles, segment  $\overline{AE}$  must bisect  $\overline{C'O}$ . That makes  $\overline{AB} = \frac{\overline{CC'}}{2} = \overline{OC} = x$ . We know  $\angle ADC'$  and  $\angle BDC$  are vertical

5 possible points

1 point (content): Realize  $A + B = 8$ .

1 point (content):  $A^2 + B^2 = 36$ .

1 point (content): Realize the need to substitute 36 into  $A^2 + 2AB + B^2 = 64$ .

1 point (content): Realize  $\frac{AB}{2} = 7$  represents the area of the triangle.

1 point (clarity): The explanation is clearly written.

115. ANSWER: 8 feet

SOLUTION: Before sliding, the situation is represented by a right triangle with hypotenuse 25 and one leg 7. Solving with the Pythagorean theorem ( $a^2 + b^2 = c^2$ , which gives  $c^2 - a^2 = b^2$ ), the other leg has to be 24 (the distance up the wall the ladder reaches).

$$\begin{aligned} 25^2 - 7^2 &= 625 - 49 \\ &= 576 \\ \sqrt{576} &= 24 \end{aligned}$$

When the ladder slides down the wall 4 feet, the vertical leg is 20 feet. With the Pythagorean theorem, the hypotenuse is 25 and one leg is 20.

$$\begin{aligned} 25^2 - 20^2 &= 625 - 400 \\ &= 225 \\ \sqrt{225} &= 15 \end{aligned}$$

Ladder was already 7 feet out, so to get to 15 it had to move 8 more feet.

3 possible points

1 point (content): Recognize the need to use the Pythagorean theorem.

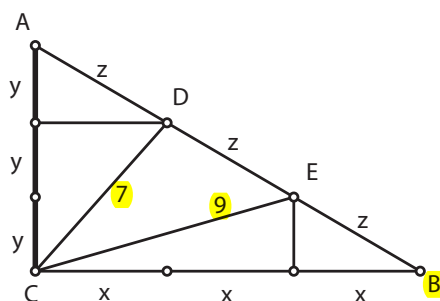
1 point (content): Algebra and arithmetic are done correctly.

1 point (clarity): The explanation is clearly written.

Bonus point for recognizing that the triangle is a 3-4-5 right triangle so, with the hypotenuse 25 and one leg 20, the missing leg has to be 15 to eliminate the calculation using the squares.

116. ANSWER:  $3\sqrt{26}$

Solution: Since the hypotenuse is trisected, parallels through those points will also trisect each leg.



From the original triangle,  $(3x)^2 + (3y)^2 = (3z)^2$  by the Pythagorean theorem.

And  $9x^2 + 9y^2 = 9z^2$  or  $x^2 + y^2 = z^2$ .

From the right triangle with hypotenuse  $\overline{CE}$ ,  $(2x)^2 + y^2 = 9^2$ .

From the right triangle with hypotenuse  $\overline{CD}$ ,  $x^2 + (2y)^2 = 7^2$ .

Adding the two gives  $5x^2 + 5y^2 = 130$ .

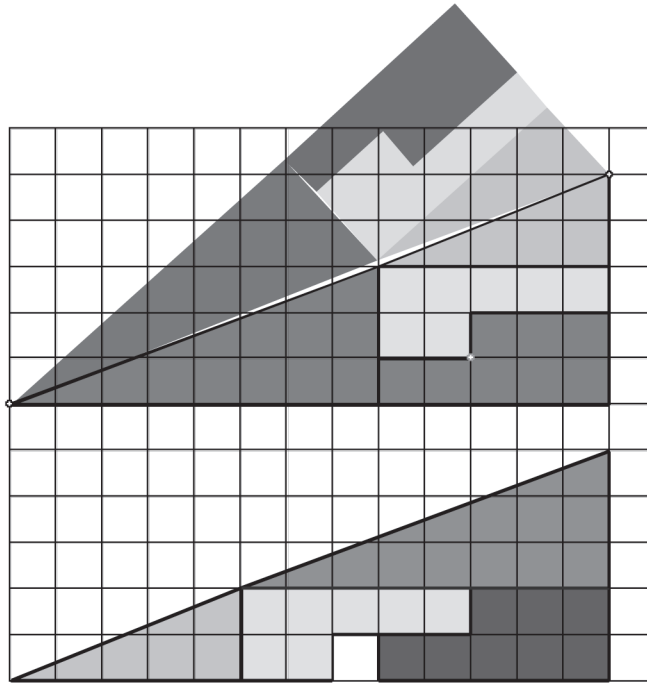
And  $x^2 + y^2 = 26$ .

So  $x^2 + y^2 = z^2$  becomes  $26 = z^2$  and  $\sqrt{26} = z$ .

$AB = 3z$  or  $3\sqrt{26}$ .

117. ANSWER: The figure is not a triangle.

SOLUTION: The smaller triangle part has a base of 5 and a height of 2. The larger triangle part has a base of 8 and a height of 3. That gives different slopes for the hypotenuses of the triangle parts when they are oriented the same. Construct the hypotenuse of the big triangle and reflect the whole shape and you will see a small region that is not shaded. This region has an area of one square unit.



4 possible points

1 point (content): Realize the large and small triangles have different slopes.

1 point (content): Realize the “hypotenuse” of the big triangle is not straight.

1 point (content): Realize that reflecting the “triangle” will show the missing area.

1 point (clarity): The explanation is clearly written.