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“If we teach children everything we know, their knowledge is limited to ours. If we teach children to think, their knowledge is limitless.”

- Michael Baker, President

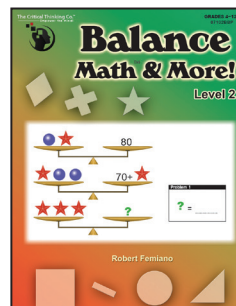
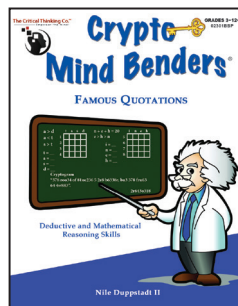
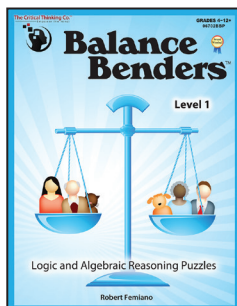
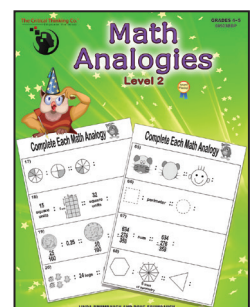
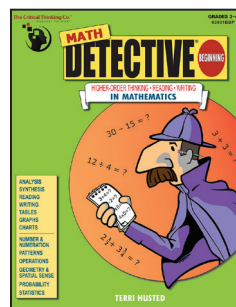
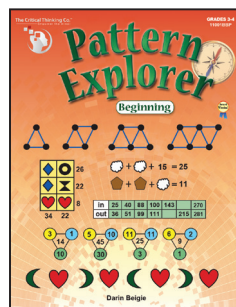
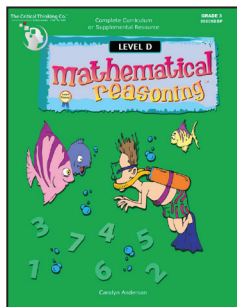
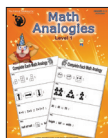


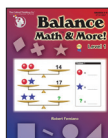
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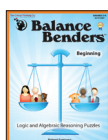
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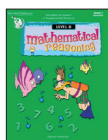
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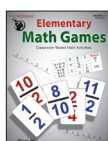
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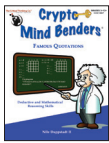
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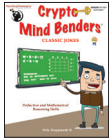
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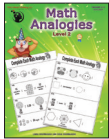
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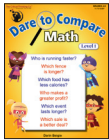
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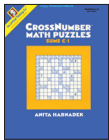
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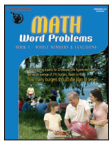
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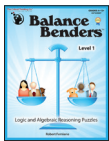
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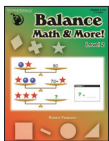
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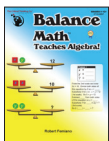
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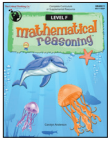
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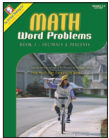
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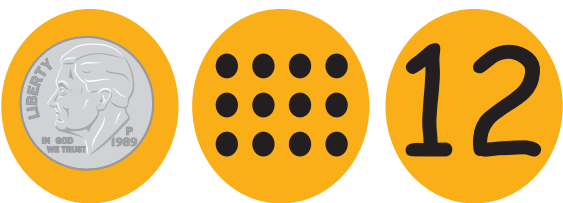
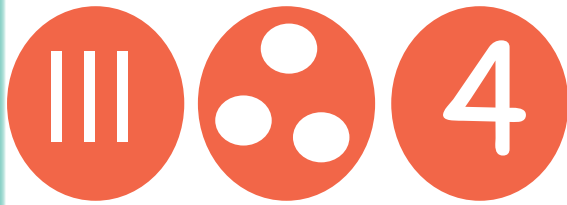
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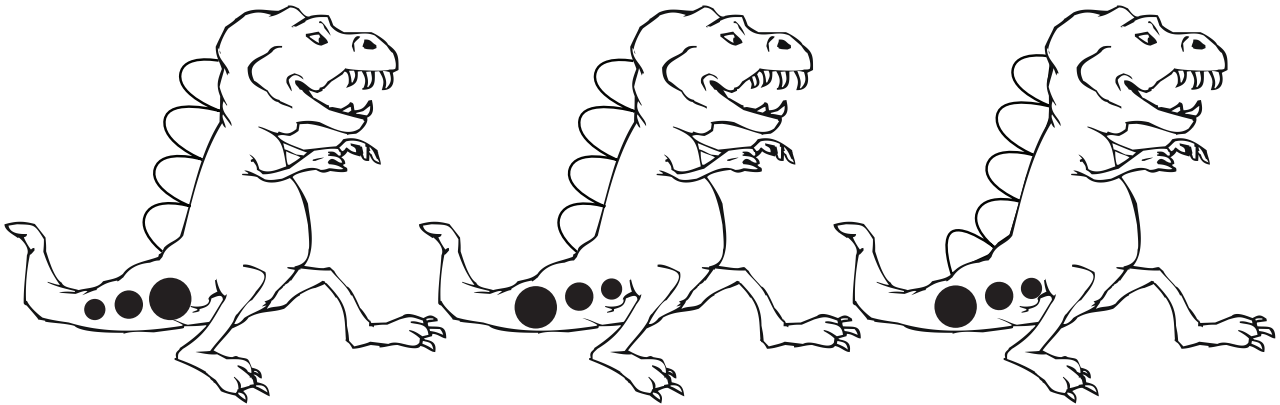
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Put an X over the thing in each set that does not belong, then explain your answer.

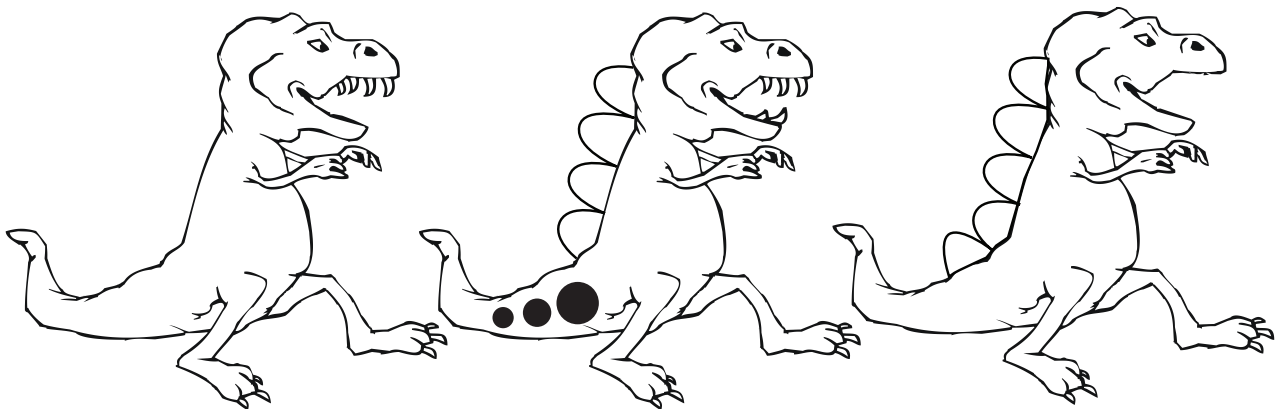


For more puzzles of this type, go to www.criticalthinking.com/mathematical-reasoning-level-b.html.

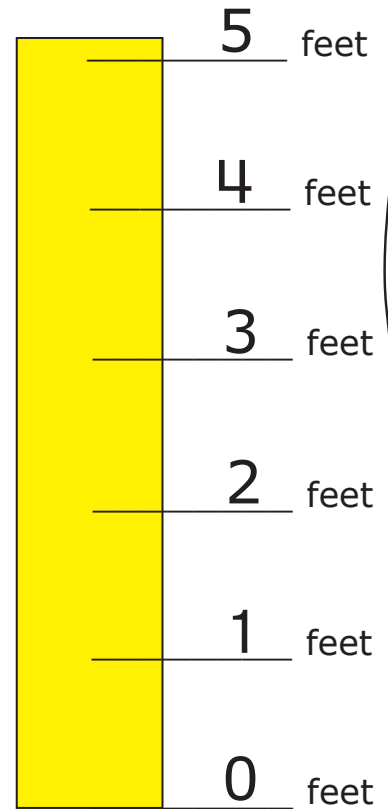
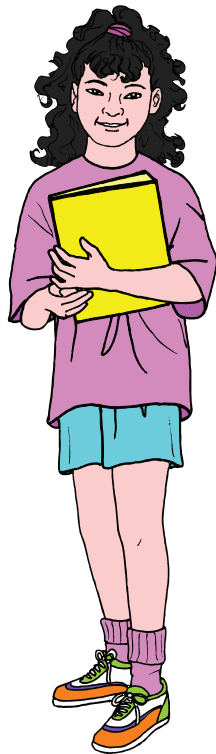
THINKER DOODLES™*



1. Look at each dinosaur above, then find its unfinished picture below. Use a pencil to draw in all the missing parts.



2. Circle the dinosaur on the bottom row with 4 upper teeth.
3. Use 2 colors to color the dinosaur on the bottom row with a small black dot at the end of its tail.
4. Use 4 colors to color all the remaining dinosaurs on the bottom row.



Use the clues and the measuring stick to find each person's name.

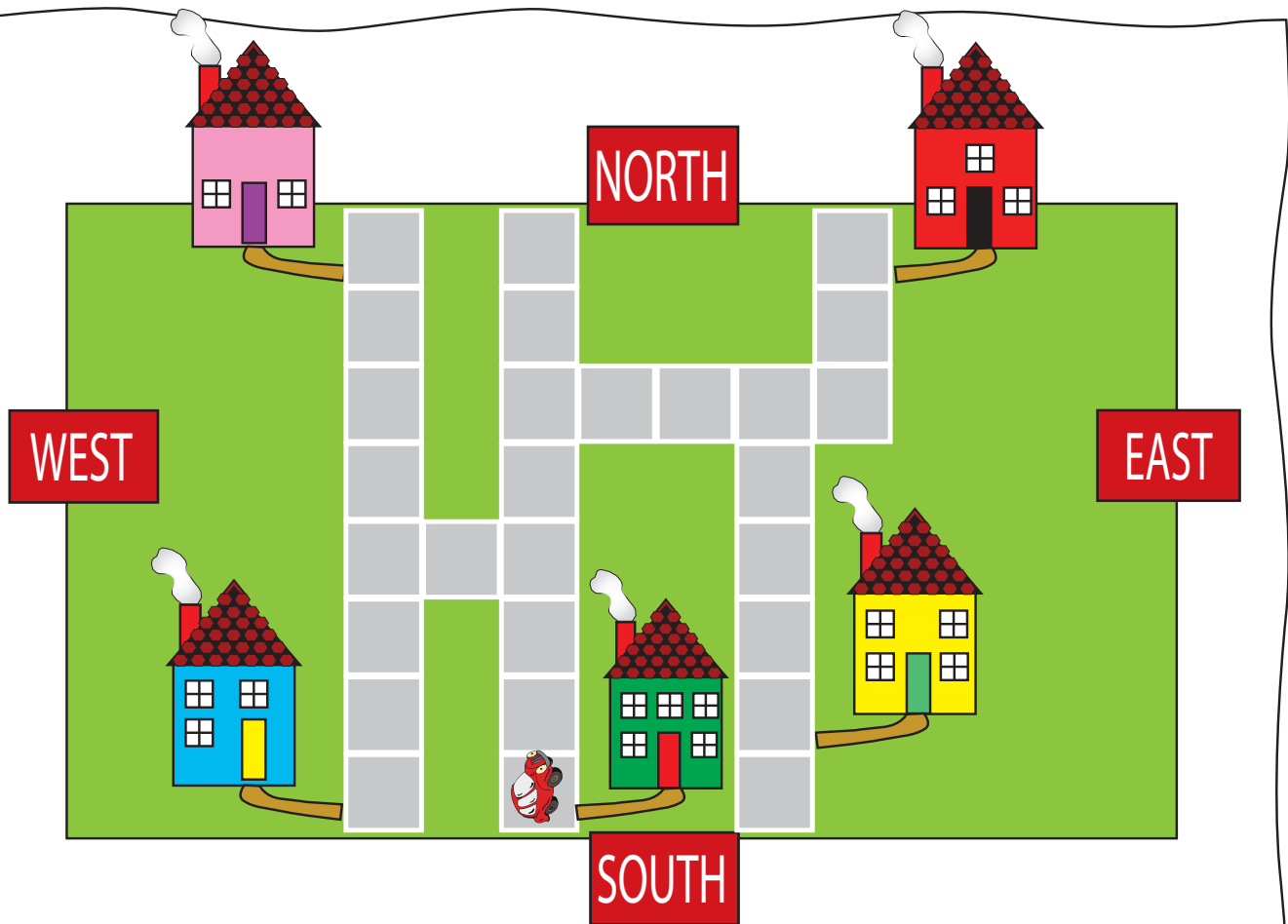
Bob is shorter than Kim. Dan has curly hair and is taller than Lee.

Use your completed diagram to write in the answers below.

Bob is _____ feet tall.

Bob and _____ are shorter than Dan.

_____ is 1 foot taller than Dan.



Give the red car road directions from the green house to the nearest house.

Go _____ squares. Then go
(compass direction) (# of squares)

_____ squares. Then go
(compass direction) (# of squares)

_____ squares. You are there.
(compass direction) (# of squares)

Give the red car road directions from the green house to the farthest house.

Go _____ squares. Then go
(compass direction) (# of squares)

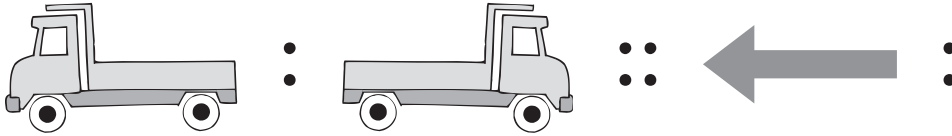
_____ squares. Then go
(compass direction) (# of squares)

_____ squares. You are there.
(compass direction) (# of squares)

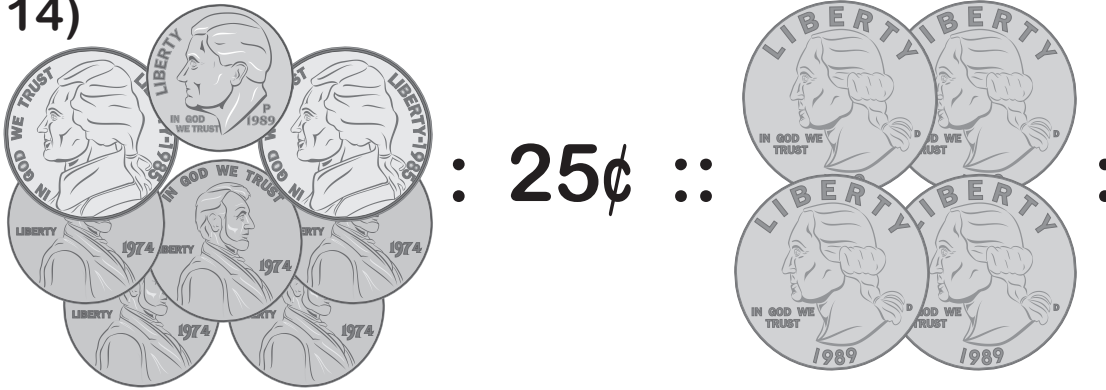


Complete Each Math Analogy

13)



14)



15)

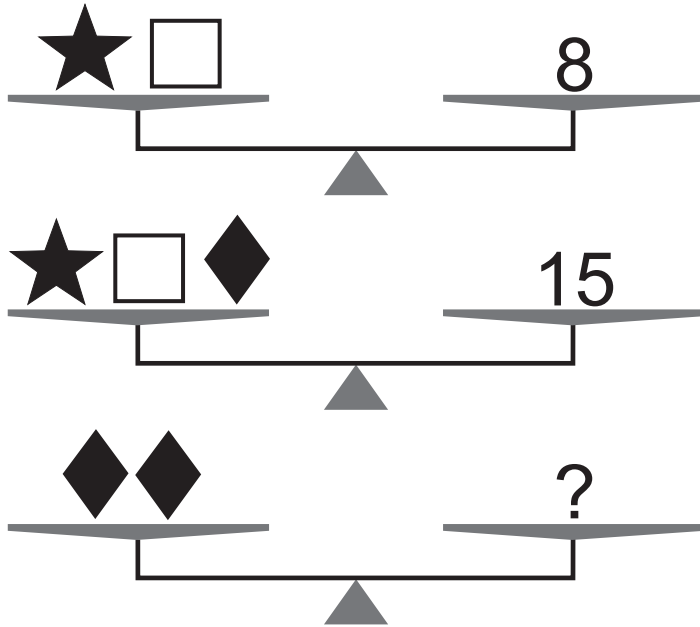
357 : 35 tens :: 246 :

16)



Balance Math

Use the balanced scales to find the missing numbers.



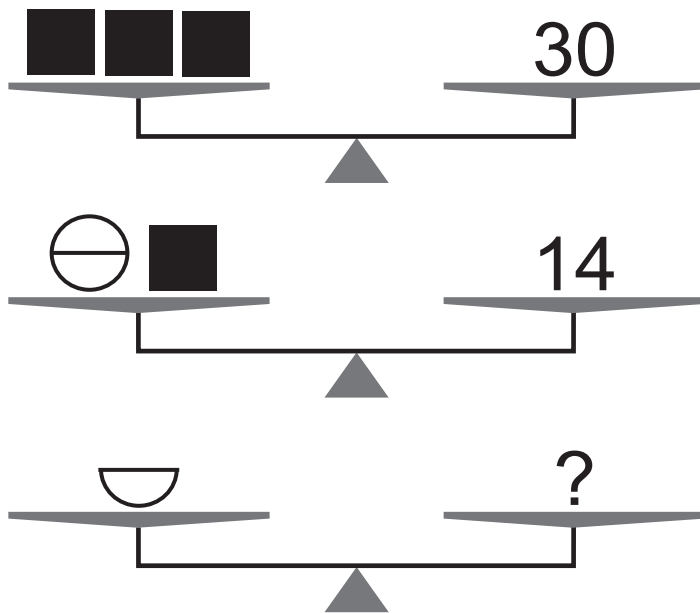
Problem 1

$\star =$ _____

$\square =$ _____

$\blacklozenge =$ _____

$? =$ _____



Problem 2

$\blacksquare =$ _____

$\ominus =$ _____

$? =$ _____

For more puzzles of this type, go to www.criticalthinking.com/balance-math-more-level-1.html.

Tic Tac Math

All rows, columns, and three numeral diagonals must add up to the same sum. Write the total and then fill in the empty spaces.

Problem 1

2	1	3
	2	1

Total:

Problem 2

6	7	
	5	
8	3	

Total:

Problem 3

4		5
6		
5		6

Total:

Problem 4

5		1
7	2	3

Total:

For more puzzles of this type, go to www.criticalthinking.com/balance-math-more-level-1.html.

INSIDE-OUT MATH

Use the clues to find the missing values.

Problem 1

a	a+c	a+d
9	18	16
b	b+c	b+d
.....	14
c	d	
.....	

Problem 2

a	a+c	a+d
.....	16
b	b+c	b+d
10	15	18
c	d	
.....	

Problem 3

a	a+c	a+d
.....	20
b	b+c	b+d
.....	16	13
c	d	
10	




Problem 4




a	a+c	a+d
.....	23	24
b	b+c	b+d
.....	22
c	d	
.....	12	




For more puzzles of this type, go to www.criticalthinking.com/balance-math-more-level-1.html.





Circle the three answers that will always be true.

a.   = 

b.   = 

c.  =  

d.     =   

e.  = 

f.   =  

For more puzzles of this type, go to www.criticalthinking.com/balance-benders-beginning.html.

NUMBER 6 MAGIC NINE

TEACHER'S INSTRUCTIONS

There are numerous magic tricks that make use of special properties of the number nine. Here is such a trick, but in this one, the number nine is well hidden.

SKILLS

- Place value (tens and ones)
- Addition facts (nine)
- Subtraction of single-digit numbers

MATERIALS

- 20 pennies

PERFORMING THE TRICK

Place twenty pennies on a table in front of your subject. Now, turn your back to the table while you give directions 1–4 below to your subject.

1. Pick up a few pennies (any single-digit number) and put them in your pocket.
2. Count the number of pennies left on the table. Sum the digits in this number.
3. Pick up the number of pennies corresponding to this sum. Put them in your pocket, also.
4. Pick up some more pennies, but hold these pennies in a closed fist in front of you. Tell me when you are done.
5. When your subject is ready, turn and face him or her while you announce how many pennies are concealed in his or her fist and add, "You can keep the eleven cents in your pocket." This announcement is quite effective if you say it immediately upon turning to your subject.

HOW THE TRICK WORKS

There will always be eleven cents in your subject's pocket. To determine the number in your subject's fist, as you slowly turn, glance at the table and note the number of pennies. As soon as you do this, look at your subject so it is not obvious that you counted the pennies on the table.

Mentally subtract this number from nine and that will be the number of pennies in your subject's fist. Thus, you can announce the number held in the closed fist and tell your subject to keep the eleven cents that was pocketed.

AN ILLUSTRATION

As your back is turned, suppose your subject picks up six pennies and places them in his or her pocket. Fourteen pennies remain on the table.

Your subject adds the digits in 14 ($1 + 4 = 5$) and removes five more pennies from the table, placing them in his or her pocket. This leaves nine pennies on the table. This is the "magic nine."

Operation Magic Tricks

Now, your subject picks up some more pennies. Suppose your subject picks up three pennies and holds them in his or her fist, as directed. This leaves six pennies remaining on the table when you turn to face your subject.

As you turn, you note the six pennies on the table; subtract 6 from 9 and announce, "You are holding three pennies in your hand. I'll take those, but you can keep the eleven cents in your pocket." You only say this if you do not mind giving away eleven cents.

WHY THE TRICK WORKS

No matter how many pennies are taken by your subject in step 1 of the trick, there will be nine pennies left on the table at the end of step 3. To prove that this is true, the table below lists every possible selection that the subject can make, from one to nine pennies.

NUMBER TAKEN IN STEP 1	NUMBER LEFT IN STEP 2	NUMBER REMOVED IN STEP 3	NUMBER ON TABLE AFTER STEP 3
1	$20 - 1 = 19$	$1 + 9 = 10$	$19 - 10 = 9$
2	$20 - 2 = 18$	$1 + 8 = 9$	$18 - 9 = 9$
3	$20 - 3 = 17$	$1 + 7 = 8$	$17 - 8 = 9$
4	$20 - 4 = 16$	$1 + 6 = 7$	$16 - 7 = 9$
5	$20 - 5 = 15$	$1 + 5 = 6$	$15 - 6 = 9$
6	$20 - 6 = 14$	$1 + 4 = 5$	$14 - 5 = 9$
7	$20 - 7 = 13$	$1 + 3 = 4$	$13 - 4 = 9$
8	$20 - 8 = 12$	$1 + 2 = 3$	$12 - 3 = 9$
9	$20 - 9 = 11$	$1 + 1 = 2$	$11 - 2 = 9$

The table above illustrates one of the special properties of the number nine. Note that nine pennies are left on the table at the end of step 3 of the trick. Of course, this means that there are eleven pennies in the subject's pocket.

Now, in step 4 of the trick, the subject takes some of the remaining nine pennies to conceal in his or her fist. As soon as you turn and see the number left on the table, nine minus this number will be the number of pennies in the subject's fist.

USING THE TRICK

The MAGIC NINE trick can be done using any set of twenty objects. You may not want your students to use a book of matches, but this works nicely.

As long as you disguise glancing at the remaining pennies on the table, your subject will have a hard time figuring out how you made your prediction. Of course, if you repeat the trick and call attention to the fact that there are eleven pennies in your subject's pocket, this will provide a clue to the secret.

The activity pages (pp. 13–14) lead the student to the discovery of the special property that leaves nine pennies on the table. When this fact is known, the student will know the prediction rule.

Operation Magic Tricks

Could more than 20 pennies be used in the trick?

THE MAGIC NINE TRICK and the MATHEMATICS CURRICULUM

Some of the special properties of nine are illustrated below in exercises from the mathematics curriculum.

1. A number is evenly divisible by 9 if the sum of the digits of the number is divisible by 9. Which of the following numbers are divisible by 9?
 - a. 504
 - b. 1519
 - c. 6345
 - d. 3692
 - e. 123,456,789
2. Given a number not divisible by 9, sum the digits of that number. Keep summing the digits of your answer until you reach a single-digit number. This number is the remainder you get if the original number is divided by 9. Try this property on the following numbers:
 - a. 1519
 - b. 3692
 - c. 1111
3. Any two-digit number minus the sum of its digits will be divisible by 9. Try this property using the following numbers:
 - a. 19
 - b. 35
 - c. 93
 - d. 27

The above property of nine was illustrated in the table on page 11.

ACTIVITY PAGES for MAGIC NINE

See if you can figure out how the magician can predict the number of pennies you are holding in your fist and that you have eleven cents in your pocket.

MAGIC NINE TRICK

Place twenty pennies on a table in front of your subject. Now, turn your back to the table while you give directions 1–4 below to your subject.

1. Pick up a few pennies (any single-digit number) and put them in your pocket.
2. Count the number of pennies left on the table. Sum the digits in this number.
3. Pick up the number of pennies corresponding to this sum. Put them in your pocket, also.
4. Pick up some more pennies, but hold these pennies in a closed fist in front of you. Tell me when you are done.
5. When your subject is ready, turn and face him or her while you announce how many pennies are concealed in his or her fist and add, "You can keep the eleven cents in your pocket." This announcement is quite effective if you say it immediately upon turning to your subject.

OBSERVATIONS

1. How many choices does the subject have when selecting pennies in step 1 in the trick?
2. Pick a number to represent one of the choices above. Using this choice, follow the next two steps in the trick and determine how many pennies remain on the table at the end of step 3.
3. Repeat observation 2 for each of the other possible choices you listed in observation 1.

Operation Magic Tricks

4. What can you conclude from the first three observations?
5. At the end of step 3 of the trick, how many pennies should the subject have in his or her pocket?

CONCLUSION

1. Use your observations above to explain how the magician is able to predict how many pennies are held in the subject's fist and how many are in the subject's pocket.

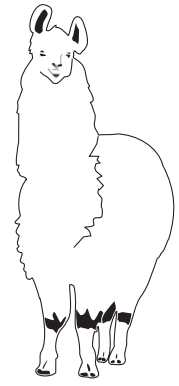
MAKING YOUR OWN TRICK

In the MAGIC NINE trick, twenty pennies were used. If more than 20 objects are used, a modification must be made in the trick. Try the trick as written but using 25 pennies. How does the trick fail? Try some other two-digit numbers greater than 20 to represent the original number of pennies. What happens in each case? Consider adding a new direction between step 3 and step 4 in the trick so the number of pennies left on the table will be nine. How many pennies would then be in the subject's pocket?

For more puzzles of this type, go to www.criticalthinking.com/operation-magic-tricks-ebook.html.

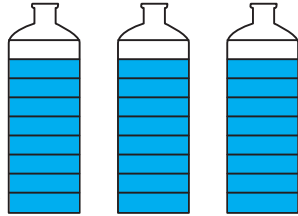
Llamas are very intelligent, and are also good pack animals. They are often used to carry water and supplies into the mountains where there are no roads.

It is a holiday and Jake is up early getting his pet llama, Larry ready for their family camping trip. Larry must carry all the water to their campsite.



Larry

One ring
equals
one gallon



Add up the number of gallons to find about how many gallons of water Larry will carry.

Write an addition number sentence for the problem.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

Complete and color the picture.



For more puzzles of this type, go to www.criticalthinking.com/mathematical-reasoning-level-d.html.

Magic Trick 1,089!



Try this.

1. Pick 3 different digits from 1, 2, 3, 4, 5, 6, 7, 8, or 9.

8, 1, 5

2. Arrange them to make the largest possible number.

851

3. Arrange them to make the smallest possible number.

158

4. Subtract the smallest number from the largest number.

$$\begin{array}{r} 851 \\ - 158 \\ \hline 693 \end{array}$$

5. Reverse the digits in the difference and add.

$$\begin{array}{r} 693 \\ + 396 \\ \hline 1,089 \end{array}$$

6. The sum is always 1,089. Try it with someone.

1 2 3 4 5 6 7 8 9

Pick 3 different digits from above. _____, _____, _____

Make the largest possible number. _____

Make the smallest possible number. - _____

Subtract. _____

Add the reversed digits. + _____

SMARTY PANTS PUZZLES™

The Friday Special!

Free candy bar with
every purchase over
\$10!



Write whether each sentence is **true**, **false**, or **unknown** based on the information on the sign.

- _____ 1. Today is Friday and Rob left the store with a candy bar and a bag of groceries so Rob had to spend over \$10 at the store.
- _____ 2. If you buy \$11 worth of candy on Friday, you can choose your own candy bar?
- _____ 3. If Tim buys \$30 on Friday, the store must give him a free candy bar.
- _____ 4. Bonnie spent \$25 at the store so she received a free candy bar.
- _____ 5. If Linda buys \$10 on Friday, the store must give her a free candy bar.

For more puzzles of this type, go to www.criticalthinking.com/mathematical-reasoning-level-d.html.

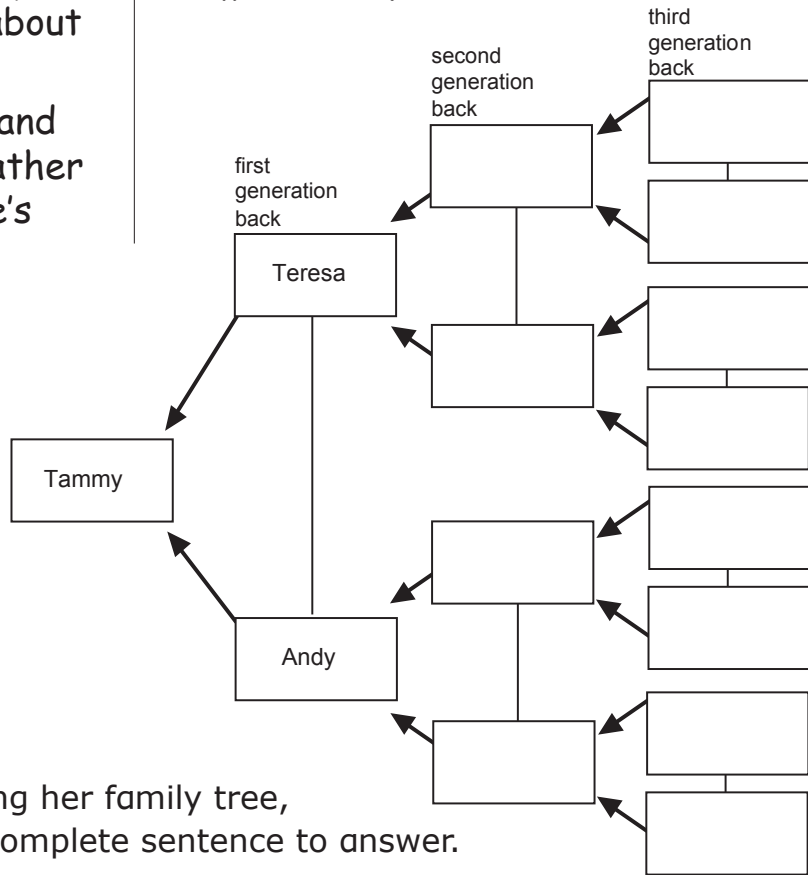
10—The Family Tree

¹All over the world, many families keep records of their ancestors. ²Ancestors are family members that came before your parents, like your grandparents, great-grandparents, etc. ³Tammy asked Grandma Emma to help her make a family tree. ⁴With her grandmother's help, Tammy was lucky enough to get the names of three generations of her family. ⁵A generation is a set of people about the same age. ⁶Tammy's parents are Teresa and Andy. ⁷Teresa's mother and father are called Emma and Joe. ⁸Joe's

parents were called Mary and Tom. ⁹Emma's mother and father were called Sofía and Antonio. ¹⁰They died when Tammy was a little girl. ¹¹Andy's parents are Angela and Camilo. ¹²Angela's mother was named Ana. ¹³Angela's father is still alive and they call him Great Grampa Mario. ¹⁴Camilo's mother was Elena, and Camilo's father was named Peter.

Questions

1. Finish Tammy's family tree:



2. Did Tammy get any help making her family tree, and if so, from whom? Use a complete sentence to answer.

Write the numbers of the two sentences that give the best evidence for the answer. _____, _____

3. How many generations, not counting Tammy 's generation, are shown on the family tree? _____
4. Who died when Tammy was a little girl? _____

Write the numbers of the two sentences that give the best evidence for the answer. _____, _____

5. Which of Tammy's great-grandparents do you know for sure is still living?

Write the number of the sentence that gives the best evidence for the answer. _____

6. Who were Teresa's grandparents on her father's side?

7. Who is Ana's and Mario's grandchild? _____

8. Continue to fill out the chart below.

	1 generations back	2 generations back	3 generations back	4 generations back	5 generations back
Tammy	Tammy's Parents	Tammy's Grandparents	Tammy's Great Grandparents	etc.	etc.
1	2	4	_____	_____	_____

Describe the pattern, using a complete sentence.

9. Tammy found one picture of each of the people in the chart above, starting with herself and going back five generations. How many pictures did she find? _____ Show your work.

For more puzzles of this type, go to www.criticalthinking.com/math-detective-beginning.html.

2. Equality Explorer 1

Each 2D shape represents a different whole number. Use the equations to find their value.

1.  +  = 18

 + 7 = 20

 = ___  = ___

2.  +  = 19

 +  +  = 24

 = ___  = ___

3.  +  = 



 + 7 = 19

 = ___  = ___

4.  +  + 15 = 25




 +  +  = 11

 = ___  = ___



5.  + 10 +  = 24

 +  = 9




 +  = 20

 = ___  = ___  = ___

6.  +  +  = 29

 +  +  = 34

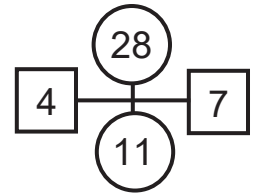
 + 13 = 28

 = ___  = ___  = ___

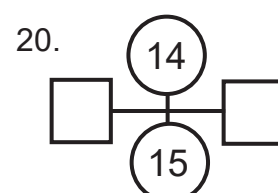
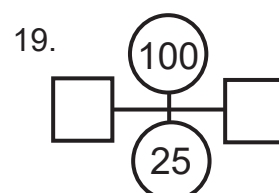
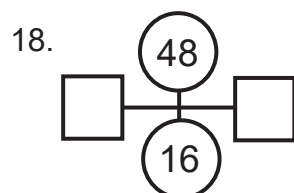
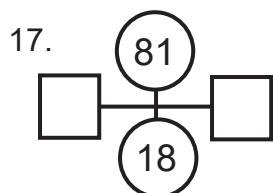
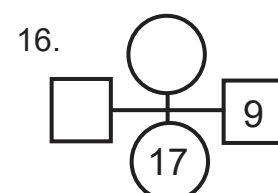
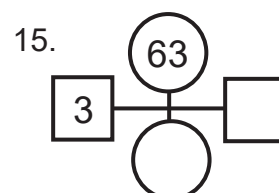
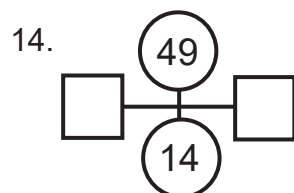
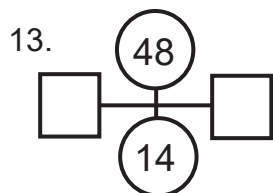
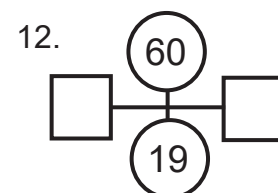
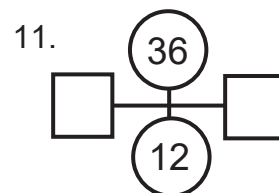
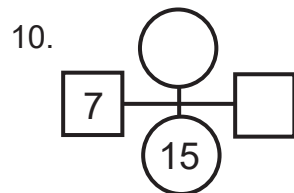
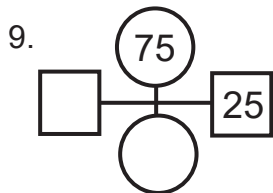
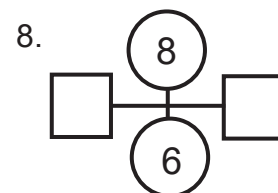
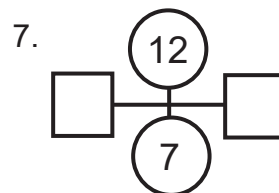
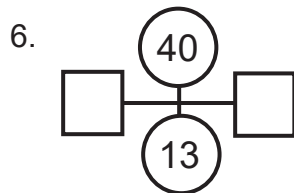
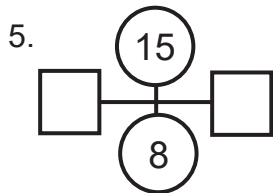
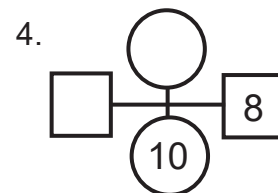
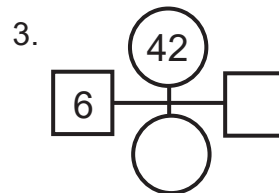
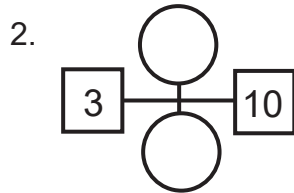
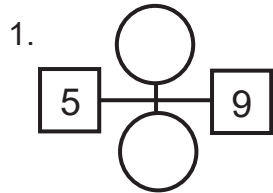
For more puzzles of this type, go to www.criticalthinking.com/pattern-explorer-beginning.html.

14. Number Ninja 3

The top circle's number equals the product of the numbers in the squares: $28 = 4 \times 7$. The bottom circle's number equals the sum of the numbers in the squares: $11 = 4 + 7$.



Fill in all missing numbers. When both squares are empty, put the larger of the two missing numbers in the right square.

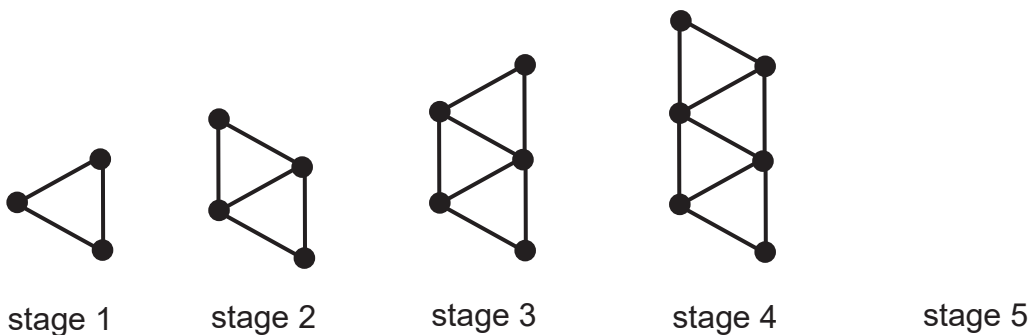


For more puzzles of this type, go to www.criticalthinking.com/pattern-explorer-beginning.html.

16. Pattern Predictor 4

The shapes below are made with toothpicks and gumdrops. For example, stage 2 has 5 toothpicks and 4 gumdrops.

1. Look at the pattern and then draw stage 5. For later stages, make a drawing if it helps you answer the questions.



2. How many toothpicks are there at stage 5?

3. How many gumdrops are there at stage 5?

4. How many toothpicks and gumdrops are there at stage 6?

• toothpicks: _____

• gumdrops: _____

5. Complete the table to show the number of toothpicks and gumdrops for stages 1 through 8.

stage	1	2	3	4	5	6	7	8
number of toothpicks		5						
number of gumdrops		4						

6. How many toothpicks and gumdrops are there at stage 12?

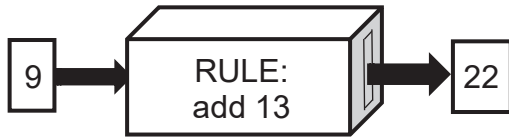
• toothpicks: _____

• gumdrops: _____

For more puzzles of this type, go to www.criticalthinking.com/pattern-explorer-beginning.html.

30. Function Finder 6

1. The function machine adds 13. So when you input 9, the output is 22. Use the rule to complete the table.



in	4	9	16	21		54
out	17	22	29		45	

2. Complete the table and state the function machine rule.

a. RULE: _____

in	14	21	29	36		74
out	24	31	39		52	

b. RULE: _____

in	6	13	18	25		57
out	28	35	40		64	

- 3a. Complete the table.

Spencer's age	5	11	23	35		66
Amanda's age	13	19	31		52	

b. Spencer is 16. How old is Amanda?

c. Amanda is 47. How old is Spencer?

- 4a. Complete the table.

cost to make cake (\$)	6	9	15	22	28	
selling price of cake (\$)	11	14	20	27		41

b. It costs \$25 to make the cake.

What is the selling price of the cake?

c. The selling price of the cake is \$17.

How much does it cost to make the cake?

- 5a. Complete the table.

Sammy's situps	20	32	45	57	70	
Sammy's pushups	5	17	30	42		88

b. Sammy does 27 situps. How many pushups does he do?

c. Sammy does 50 pushups. How many situps does he do?

For more puzzles of this type, go to www.criticalthinking.com/pattern-explorer-beginning.html.

Game 4

CAN YOU MAKE.....?

Materials

- Pencil and paper
- Index cards
- Timer
- Overhead, whiteboard, or Smartboard (optional)

Learning Standards for Mathematics

- Fluency using the four operations
- Generate and analyze patterns
- Perform operations with multi-digit whole numbers
- Interpret numerical expressions

– Overview –

Students use number sense, operational skills, and strategies to compute a target number from five randomly chosen numbers. Operational fluency and precision is an asset in this activity.

– The Game –

Distribute six index cards to each student. Each student randomly picks five numbers from 1 to 25 (or from any appropriate interval for the class level) and writes down each number on a separate card. The sixth card is for keeping score. Choose any number from 1 to 50 (or from any interval that is appropriate for your class level) and write it on the board. To involve the students in choosing the number, pick any student, ask the number of the day of her/his birthday, and use that number.

Students have an allotted time (about 1 to 1.5 minutes) to use at least two numbers from their cards and any operation (addition, subtraction, multiplication, and division) to help compute the number on the board. A student must use at least two numbers from her/his cards to compute the number. The same number may not be used twice in any one computation.

As soon as a student has an answer, they raise their hand. When the allotted time is up, students with raised hands will be called on to explain their answers. A correct answer gains one point.

Students should have an index card on which to keep track of their score. This is a good time to use tallying. As students accumulate their scores, explain that keeping score in groups of 5 is so much easier when computing the final score. Look at the difference for a score of 12: ||||| ||||| or III III II.

Example:

The numbers on my cards are 5, 10, 11, 13, 7

The number on the board is 25.

Possible answer: $13 + 7 + 10 - 5 = 25$

The amazing outcomes – Every number on the board will have at least one solution within the class!

Questions for Further Discovery

- 1 Based on the interval chosen for the cards, for example 1 to 25, what is the highest target number you could compute?

$$25 \times 24 \times 23 \times 22 = 303,600.$$

- 2 If the number cards only have odd numbers, what computations must you do to get an even result?

Some suggestions might be:
Add an even amount of your odd numbers
Subtract two odd numbers.

- 3 If the number cards only have even numbers, is it possible to get an odd result?

Yes, if you use division.
Even + Even = Even
Even - Even = Even
Even \times Even = Even
Even \div Even = odd or even depending on the numbers.

$$12 \div 6 = 2 \quad 12 \div 4 = 3$$

See "Teachable Moments" (page 133) for further study of odd and even numbers.

Activity 29

Use the clues and the chart to determine the value of each letter, solve the cryptogram, and discover the famous quote.

$10\% \text{ of } 100 = b$
 $6\% \text{ of } 300 = e + b$
 $e - (s + 1) = h$
 $s < h$

	e	h	s	b
10				
8				
5				
2				

$e = \underline{\quad}$
 $h = \underline{\quad}$
 $s = \underline{\quad}$
 $b = \underline{\quad}$

$(11\% \text{ of } 400) - t = 32$
 $t \div o = u \times 3$

	c	o	u	t
12				
7				
4				
1				

$c = \underline{\quad}$
 $o = \underline{\quad}$
 $u = \underline{\quad}$
 $t = \underline{\quad}$

$r \neq 3$
 $n = 44 - 38$
 $r \times w \geq 27$
 $r \times w < 28$

	a	w	r	n
11				
9				
6				
3				

$a = \underline{\quad}$
 $w = \underline{\quad}$
 $r = \underline{\quad}$
 $n = \underline{\quad}$

Cryptogram (Parentheses separate double digits; they have no other meaning.)
 "(10)8 (12)58 75(11)6g8 y14 3(11)6(12) (12)1 288 i6
 (12)58 319ld."

M(11)5(11)(12)m(11) G(11)6d5i

" _____ g _____ y _____
 _____ i _____ ld."
 M _____ m _____ G _____ d _____ i

Activity 3

Use the clues and the chart to determine the value of each letter, solve the cryptogram, and discover the classic joke.

$n - s = f$
 $f < 5$

	f	n	s	i
1				
2				
5				
6				

f = ____
 n = ____
 s = ____
 i = ____

$a + t = 12$
 $a + h = 19$

	a	t	h	r
3				
4				
9				
10				

a = ____
 t = ____
 h = ____
 r = ____

$p = c + 4$
 $m = c + 1$

	p	e	c	m
7				
8				
11				
12				

p = ____
 e = ____
 c = ____
 m = ____

Cryptogram (Parentheses separate double digits; they have no other meaning.)

W(10)93 25 9 14og'5 19vo423(12) 8u527? (10)2(11)
 (10)o(11)!

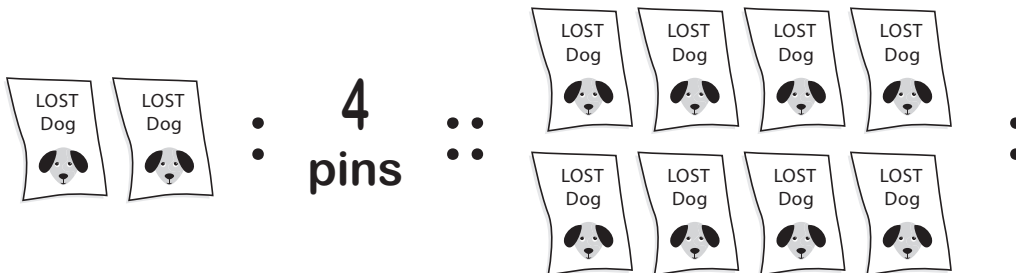
W _ _ _ _ _ _ _ _ _ _ og' _ _ _ _ vo _ _ _ _ _
 _ u _ _ _ ? _ _ _ _ _ o _ !

For more puzzles of this type, go to www.criticalthinking.com/crypto-mind-benders-classic-jokes.html.



Complete Each Math Analogy

5)



6)

2,000,000 : $\frac{4,000,000}{6,000,000}$:: 4,000,000 :

8,000,000

7)



8)

dinner : leftovers :: payment :

25. Who travels the longest distance? Who travels the shortest distance?

Tanya walks
4 feet every
second for
60 seconds.



Kevin cycles
30 feet every
second for
7 seconds.

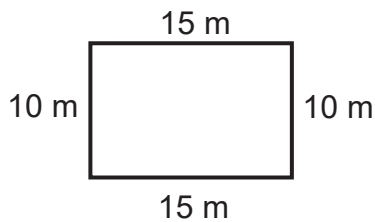


Amanda runs
9 feet every
second for
25 seconds.

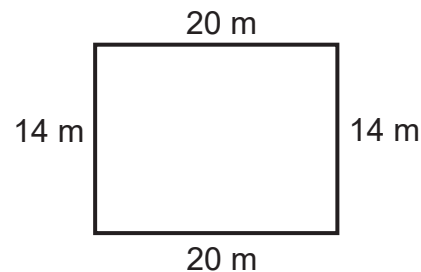


26. Joanne and Remy each build a wooden fence around their rectangular vegetable garden. Whose fence costs more?

Joanne's fence costs \$7 per meter.



Remy's fence costs \$5 per meter.



27. In the list below, each person's heart beats at a constant rate. Whose heart will have the most beats in 1 minute? Whose heart will have the fewest beats in 1 minute?

- Adam's heart beats 25 times in 20 seconds.
- Rachel's heart beats 160 times in 120 seconds.
- Brett's heart beats 18 times in 15 seconds.



For more puzzles of this type, go to www.criticalthinking.com/dare-to-compare-math-level-1.html.

37. Which is a longer amount of time?

a. 450 minutes or 7 hours

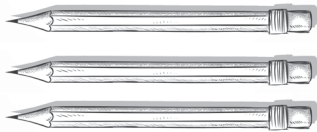
b. 1,000 seconds or 20 minutes

1 hour = 60 minutes
1 minute = 60 seconds

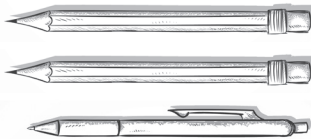
c. 190 minutes or $3\frac{1}{3}$ hours

d. 4,000 seconds or 1 hour

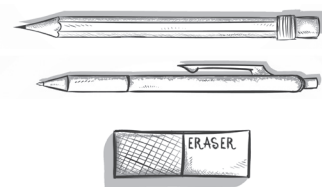
38. What is the price of a pencil? What is the price of a pen? What is the price of an eraser?



3 pencils cost 60¢



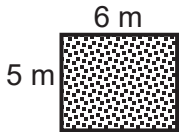
2 pencils and
1 pen cost 75¢



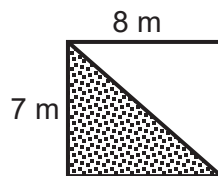
1 pencil, 1 pen, and
1 eraser cost \$1.15

39. Which shaded region has the largest area? Which shaded region has the smallest area?

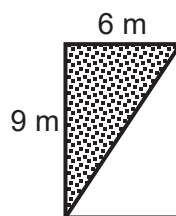
A



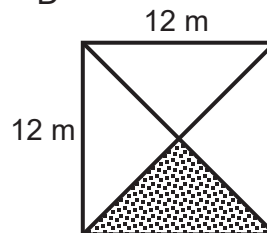
B



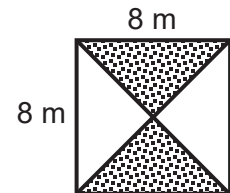
C



D



E



For more puzzles of this type, go to www.criticalthinking.com/dare-to-compare-math-level-1.html.

103. Whose car has more gas left after their trip?

Flora's car starts with 11 gallons of gas and she drives 150 miles.



Flora's car travels 30 miles for every gallon of gas.

Mason's car starts with 14 gallons of gas and he drives 175 miles.



Mason's car travels 25 miles for every gallon of gas.

104. Whose meal has more calories?

Jonah's meal

- 8 ounces of chicken
- 3 ounces of rice
- 6 ounces of beans

Camilla's meal

- 6 ounces of chicken
- 6 ounces of rice
- 5 ounces of beans

Nutrition Information

- 4 ounces of chicken has 200 calories
- 6 ounces of rice has 240 calories
- 2 ounces of beans has 60 calories

105. Whose jar holds more water when full?



Nora's jar is $\frac{2}{3}$ full, but if she adds 6 more cups it will be completely full.



Robin's jar is $\frac{3}{4}$ full, but if she adds 5 more cups it will be completely full.

For more puzzles of this type, go to www.criticalthinking.com/dare-to-compare-math-level-1.html.

- A number below a diagonal line shows the sum for the squares underneath.
- A number above a diagonal line shows the sum for the squares to the right.
- You may use only the digits 1 through 9 (one digit per square).
- You may not use any digit more than once to get a sum.

(Three answer digits are given.)

13.

	16	11		13	10		13	10	4	13	16
10			13			16					
			15				15				
15						20					
7				10					17		
								12			
6		3		11			7			16	14
			15			13					
	16				16			24			
	17	13			11			17			
16			17			15			14		
			16			7			20		
21				3			11				
			13				26			17	14
	6	7	13			11		10			
						15					
6			12					14		8	
			6					15			
15						35	8				
16						16				3	

- 95.** A dozen eggs will make four omelets.
- a–c.** How many eggs are needed to make
 - a.** 8 omelets?

 - b.** 1 omelet?

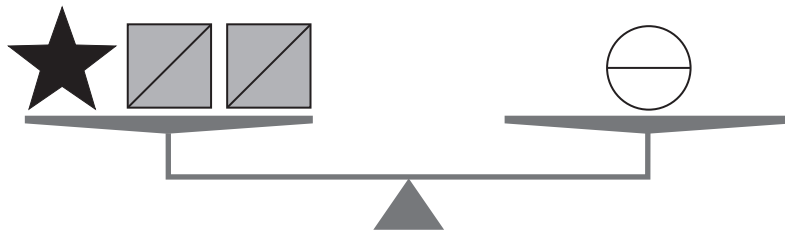
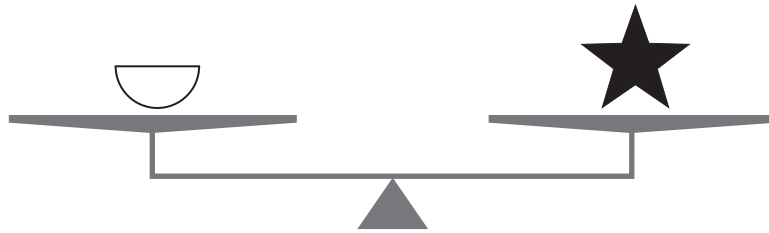
 - c.** 9 omelets?

 - d–f.** How many omelets can be made from
 - d.** 2 dozen eggs?

 - e.** 9 eggs?

 - f.** 21 eggs?
- 96.** A concrete pipe has an outside diameter of 20 inches. The pipe is 2 inches thick. What is its inside diameter?

Balance Benders™



Circle the three answers below that will always be true.

a. = d. =

b. = e. =

c. = f. =

For more puzzles of this type, go to www.criticalthinking.com/balance-benders-level-1-ebook.html.

Tic Tac Math

All rows, columns, and three numeral diagonals must add up to the same sum. Write the total and then fill in the empty spaces.

Problem 1

45		
	55	
	75	65

Total:

Problem 2

111		124
136		
122		

Total:

Problem 3

	80	
40	120	140

Total:

Problem 4

51	9	39
	33	

Total:

For more puzzles of this type, go to www.criticalthinking.com/balance-math-more-level-2.html.

INSIDE-OUT MATH

Use the clues to find the missing values.

Problem 1

a	a-c	a-d
b	b-c	b-d
	c	d

Problem 2

a	a-c	a-d
b	b-c	b-d
	c	d

Problem 3

a	a-c	a-d
b	b-c	b-d
	c	d

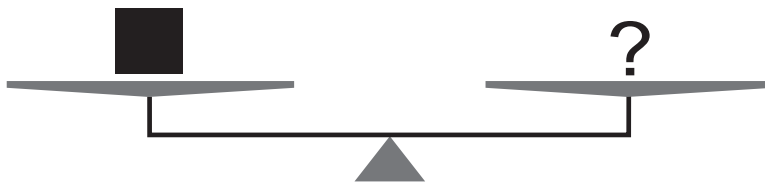
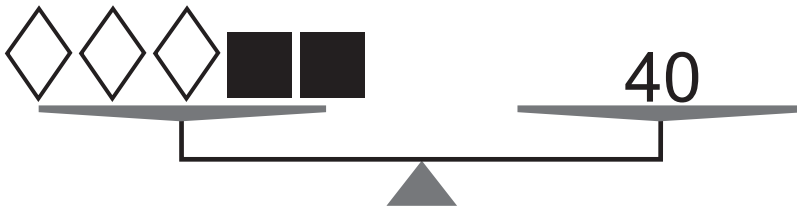
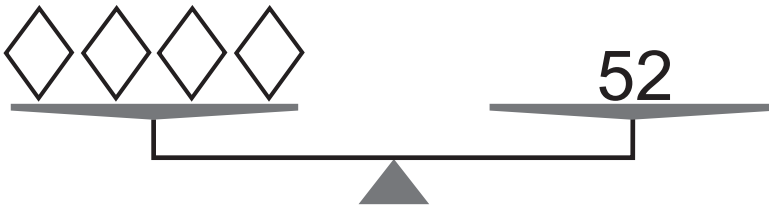
Problem 4

a	a-c	a-d
b	b-c	b-d
	c	d

For more puzzles of this type, go to www.criticalthinking.com/balance-math-more-level-2.html.

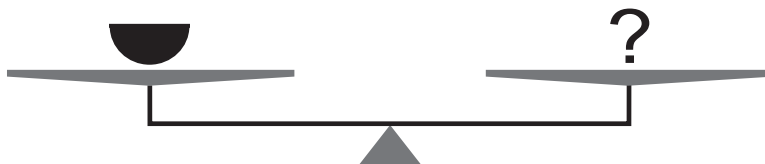
Balance Math

Use the balanced scales to find the missing numbers.



Problem 1

? =

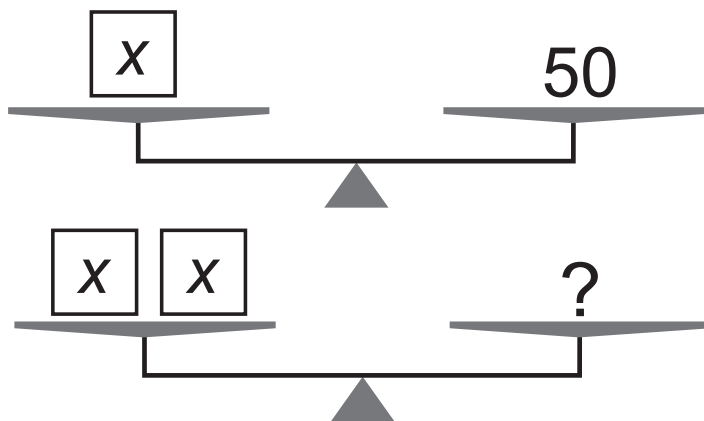


Problem 2

? =

For more puzzles of this type, go to www.criticalthinking.com/balance-math-more-level-2.html.

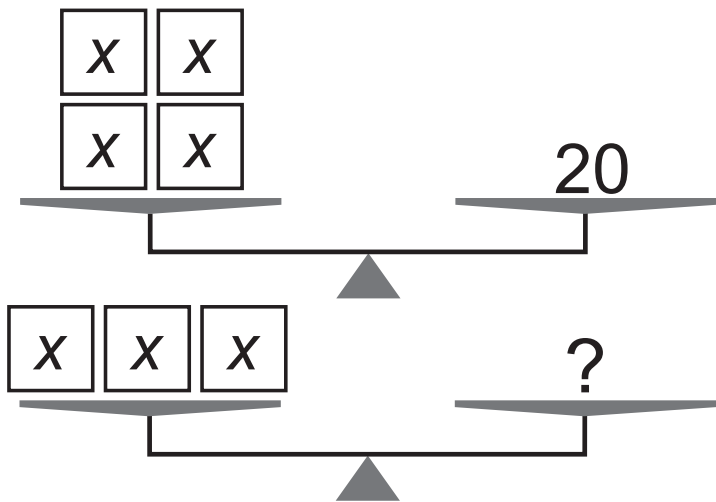
Write the answers in the box, then fill in the blanks in the proof.

**Problem 1**

$$X = \dots\dots\dots$$

$$? = \dots\dots\dots$$

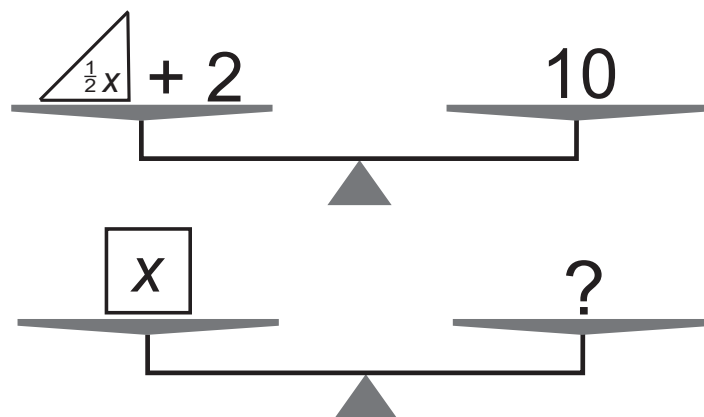
From the 1st scale we know: $x = 50$.
 Substitute $\underline{\quad}$ for x in: $2x = ?$ (2nd scale).
 So $2(\underline{\quad}) = 100 = ?$

**Problem 2**

$$X = \dots\dots\dots$$

$$? = \dots\dots\dots$$

From the 1st scale we know: $4x = 20$.
 Divide both sides of this equation by 4
 so $x = \underline{\quad}$. Substitute $\underline{\quad}$ for x in:
 $3x = ?$ (2nd scale). So $\underline{\quad}(5) = 15 = ?$

**Problem 3**

$$\frac{1}{2}X = \dots\dots\dots$$

$$? = \dots\dots\dots$$

From the 1st scale we know: $\frac{1}{2}x + 2 = 10$.
 Subtract $\underline{\quad}$ from both sides of this
 equation so $\frac{1}{2}x = \underline{\quad}$. Substitute $\underline{\quad}$ for
 each $\frac{1}{2}x$ in: $x = ?$ (2nd scale).
 (Note: $1x$ is the same as x .)
 So $\underline{\quad} + \underline{\quad} = \underline{\quad} = ?$

Happy Birthday



Joan's tenth birthday is today. Her friend Lupe was born on the same day, but one year later. Joan's teacher Mrs. Bole led the school in a happy birthday song for Joan and Lupe. Lupe's teacher Miss Tels, told Lupe that she was 78,840 hours old. They both laughed when Lupe told her she didn't realize she was that old.



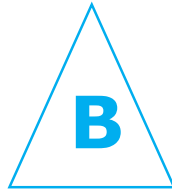
Fill in the chart.

Measure of Time	Joan	Lupe
Years Old		
Days Old (Years \times 365)		
Hours Old (Days \times 24)		
Minutes Old (Hours \times 60)		

For more puzzles of this type, go to www.criticalthinking.com/mathematical-reasoning-level-f.html.



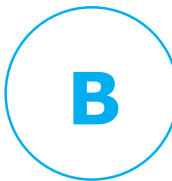
Use a pencil and eraser to find the objects in the bottom row that will satisfy all of the clues. The solution will have a triangle, a circle, and a square.



One of these triangles is in the bottom row, and it is in the same column.



One of these squares is in the bottom row, and it is in the same column.



One of these circles is in the bottom row, and it is in the same column.



Two of these figures are in the bottom row, but only one is in the same column.

Smarty Pants Puzzles™*

Zimmerman Mouthwash

This mouthwash is the best mouthwash you can buy! It has natural ingredients to fight germs that cause bad breath. Zimmerman mouthwash has been recommended by doctors for severe cases of bad breath. After just one capful of Zimmerman mouthwash, we guarantee your breath will smell Zimmerman fresh—or the Zimmerman Company will give you your money back!



Read the advertisement and accept what is written as true. Write whether each sentence is true (**T**), false (**F**), or unknown (**U**) based on the information.

_____ Zimmerman Mouth Wash works better than other types of mouthwash you can buy.

_____ Zimmerman Mouthwash “is” recommended by doctors?

_____ If Zimmerman Mouthwash doesn’t cure your bad breath, the ad says that the Zimmerman Company will give you your money back.

_____ Zimmerman will kill germs that cause bad breath.

_____ The ad says that Zimmerman Mouthwash is made from all natural ingredients.

For more puzzles of this type, go to www.criticalthinking.com/mathematical-reasoning-level-f.html.

1—The Train Ride

¹Mrs. Applecrumb, Mrs. Winterbloom, Mr. Papas, Ms. Kamen, Ms. Twinkle, and Mr. Lyons were in the same train car. ²Mrs. Applecrumb sat in seat 10. ³Mrs. Winterbloom sat in seat 5 next to Mr. Papas, who sat in seat 6. ⁴Ms. Kamen always sat in the back on the last seat, seat 27, because she liked to stretch her legs and grade her students' papers. ⁵Ms. Twinkle sat in seat 20 next to Mr. Lyons, who sat in seat 19.

⁶The train engineer loved to play math games during the ride. ⁷He said over the loud speaker, "Does anyone know what a factor is?" ⁸Mrs. Winterbloom, who was not really listening, said, "This man is silly. ⁹Of course, I know what a tractor is." ¹⁰Mr. Papas laughed and explained to her that the word was "factor" not "tractor." ¹¹The train

engineer then said, "If any of you are seated in a seat that is a factor of 20, you will get a free engineer's cap. ¹²Now remember, my fellow riders, a factor is any number that goes into another number evenly or without a remainder." ¹³He explained, "Remember, the factors of 12 are 1, 2, 3, 4, 6, and 12. ¹⁴Don't get this confused with the word m-u-l-t-i-p-l-e (he said this word very slowly). ¹⁵The multiples of 12 are 12, 24, 36, 48, 60, etc., etc." ¹⁶Mr. Lyons became very happy. ¹⁷He began to shout, "I am seated on a seat that is a factor of 20." ¹⁸This made Ms. Kamen very upset. ¹⁹She began to mumble to herself and shake her head.

²⁰Then the engineer said, "By the way, any of you who are seated in a seat that is a multiple of 1 will get a free ride next time."

Questions

1. What are the factors of 20? _____

Give the number of the sentence that provides the best evidence for the answer.

2. Of the people mentioned in the story, who will get a free engineer's cap on this ride? Use complete sentences to explain your thinking.


3. Is Mr. Lyons correct in thinking that he will be getting a free engineer's cap on this ride? Why or why not? Use complete sentences to explain your thinking.

4. Which of the following shows the first four multiples of 3?

- a. 3, 6, 9, and 12
- b. 3, 9, 12, and 18
- c. 1, 3, 6, and 9
- d. none of these

Give the number of the sentence that provides the best evidence for the answer.

5. Of the people mentioned in the story, who is seated in a seat that is a multiple of 3?

6.  Why is 1 a factor of every number? Use complete sentences to explain your thinking.

7. Who will be getting a free ride next time they ride the train?

- a. Ms. Kamen
- b. Mr. Papas and Ms. Twinkle
- c. Mrs. Applecrumb
- d. all of them

For more puzzles of this type, go to www.criticalthinking.com/math-detective-a1.html.

56. You clip a coupon from the newspaper which offers you a \$1 refund from the manufacturer if you mail it to them with a label from their product. You buy the product for \$3.23 plus 4% sales tax. You mail the coupon and label to the manufacturer at a cost of 15¢ for the stamp and 1¢ for the envelope. The manufacturer sends you a check for \$1.
- How much did it cost you to send the coupon to the manufacturer?
 - How much profit did you make from sending the coupon to the manufacturer?
 - How much sales tax did you pay on the product?
 - How much (total) did you pay for the product at the store?
 - What percent of the total you paid was your profit?
 - How much did the product end up costing you?
 - What percent of the original sale price (including tax) was your actual cost of the product?

For more puzzles of this type, go to www.criticalthinking.com/math-word-problems-book-2-ebook.html.

THE TRICK: Magical Number Nine

Variation One

1. Select a three-digit number in which none of the three digits is the same.
2. Form another three-digit number by rearranging the original digits.
3. Now subtract the two three-digit numbers and add the digits in this difference.
4. From the first (or last) digit in the sum, you can tell the other digit.

Variation Two

1. Follow the steps above through step #3.
2. If the sum is not one digit, continue adding the digits in the result until you reach a one digit number. You can predict this digit.

UNRAVELING THE TRICK

1. Choose three different variables for the hundred's, ten's and unit's digit of the number, and write them in appropriately labelled columns.
2. Borrow one from the hundred's column, and rewrite the hundred's and ten's digits.
3. Borrow one from the ten's column, and rewrite the ten's and unit's digits.
4. Suppose the permutation selected is the original digits in reverse order. Subtract these digits from those appearing in the table.
5. Sum the digits in the difference above, and use this result to explain the trick.

QUESTIONS TO PONDER

1. What would happen if all the digits in the chosen number were the same?
2. Can two of the digits be the same?
3. Can you verify the trick if one uses a permutation other than the original digits reversed?
4. Will the sum of the digits in the difference always be a multiple of 9?

For more puzzles of this type, go to www.criticalthinking.com/algebra-magic-tricks-book-1-ebook.html.

THE TRICK: Who's Got What?

1. Select three people as subjects. Call them P , Q , and R .
2. Place three items on a table. Call them a , b , and c .
3. While your back is turned, ask each person to select one of the items and conceal it.
4. Place 24 counters (you can use poker chips) on the table. Give one chip to Q and two chips to R . Person P gets none.
5. With your back turned, ask whoever selected item a to take as many chips as s/he now has, whoever took b to take three times as many chips as s/he now has, and whoever has c to take nine times as many chips as s/he now has.
6. Face the subjects and ask one of them to carry out the following divisions with a number you suggest:
 - Divide the suggested number by 3, noting the quotient and remainder.
 - Divide the quotient by 3, noting the new quotient and remainder.
 - Divide the new quotient by 3, noting the quotient and remainder.
7. Now you can announce who is concealing each of the items.

UNRAVELING THE TRICK

1. How many ways can P , Q , and R make selections from the items a , b , and c ?
2. Choose one of the possible selections found above and note the number of counters given to the subjects (3), the number taken by the subjects (call this number N), and the number left on the table (call this T). How can you determine the value of N if you know the value of T ?
3. For the selection used above, carry out the three divisions (in step #6 of *THE TRICK*) using the number N as your first dividend. What are the three remainders?
4. Let the numbers 0, 1, and 2 be associated with the three subjects P , Q , and R , respectively ($P = 0$, $Q = 1$, $R = 2$). Compare your three remainders (found in #3 above) with the identities of the persons who selected items a , b , and c . Can you predict who took each item?

QUESTIONS TO PONDER

1. Does the number of counters used in the trick have to be 24?
2. What is the minimum number of counters needed in the trick?
3. How many remainders are possible when a given number is divided by 3?
4. After the three divisions are carried out, will any two remainders be equal?

FURTHER INVESTIGATIONS

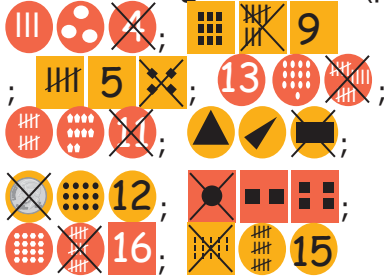
1. Suppose there were four subjects (P , Q , R , and S) selecting from four items (a , b , c , and d). Also, suppose you gave 1 counter to Q , 2 to R , 3 to S , and 0 to P . Determine how you would alter the trick so that you could predict which subject selected each item. (Hint: Have a subject take either the same number of counters, 4 times as many, 16 times as many, or 64 times as many as s/he has.)
2. Show how converting the number of counters taken by the subjects to a base four numeral could be used to predict who took what.
3. If there were 5 subjects, what is the minimum number of counters that would be needed? How could you alter the trick to solve the problem of needing large numbers of counters for your prediction?

For more puzzles of this type, go to www.criticalthinking.com/algebra-magic-tricks-book-2-ebook.html.

Answers

Mathematical Reasoning™ Level B (pp. 1-4)

Page 1



Page 2



Coloring will vary.

Page 3

Bob is 2 feet tall. Bob and Lee are shorter than Dan. Kim is 1 foot taller than Dan.

Page 4

north, 3; west, 2; south, 3; north, 5; east, 3; south, 4
(do not count the space the car was on.)

Math Analogies Level 1 (p. 5)

13)

14) 100¢

(any dollar value is acceptable)

15) 24 tens

(4 is in the 10s place of 246, but there are 24 tens in 246. Think of money and you have 24 dimes and 6 pennies.)

16)



(any object with a fourth shaded is acceptable)

Balance Math™ & More! Level 1 (pp. 6-8)

Page 6

Problem 1: ★ □ = 8, ◆ = 7, ? = 14

Explanation: Substitute 8 for ★ □ on 2nd

balance so 8 + ◆ = 15. Remove 8 from both sides so

◆ = 7. ◆ ◆ = 7 + 7 = 14.

Problem 2: ■ = 10, ⊖ = 4, ? = 2

Explanation: Divide both sides on 1st balance in thirds so ■ = 10. Substitute 10 for ■ in 2nd balance so 10 + ⊖ = 14. Remove 10 from both sides so ⊖ = 4. Divide in half so ∪ = 2.

Page 7

Problem 1:

2	1	3
3	2	1
1	3	2

Total: 6

Problem 2:

6	7	2
1	5	9
8	3	4

Total: 15

Problem 3:

4	6	5
6	5	4
5	4	6

Total: 15

Problem 4:

5	6	1
0	4	8
7	2	3

Total: 12

Page 8

Problem 1: b = 7
c = 9
d = 7
b + c = 16

Problem 2: a = 8
c = 5
d = 8
a + c = 13

Problem 3: a = 10
b = 6
d = 7
a + d = 17

Problem 4: a = 12
b = 11
c = 11
b + d = 23

Balance Benders™ Beginning (p. 9)

Page 9: c, d, e

c. Remove ■ from both pans. (Tip 4)

d. Add ■ to both pans and reverse. (Tips 3 and 1)

e. Divide = in half so = (Tip 6)

Operation Magic Tricks (pp. 10-14)

Refer to the activity.

Mathematical Reasoning™ Level D (pp. 15-17)

Page 15

8 + 8 + 8 = 24 gallons

Page 16

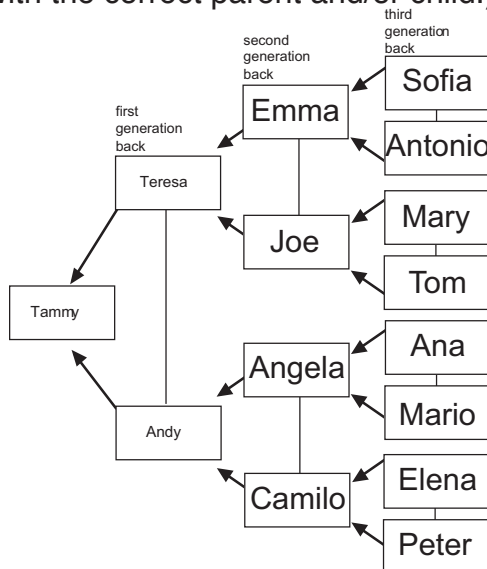
Answers will vary.

Page 17

- unknown (He may have spent less than \$10 but bought the candy bar.)
- unknown (The sign did not say you could pick your candy bar.)
- true
- unknown (Did she spend the money on Friday?)
- false (The sign said "over" \$10.)

Math Detective® Beginning (pp. 18-19)

- (Note: spouses can be given in reversed order as long as each person remains with the correct parent and/or child.)



- Yes, her grandmother Emma helped her. Sentences 3 and 4
- 3
- Emma's mother and father OR Sofia and Antonio, Sentences 9 and 10
- Great Grampa Mario, Sentence 13
- Mary and Tom (See sentences 7 and 8.)
- Andy
- 8, 16, 32, Each number is doubled from the one before.
- $63, 1 + 2 + 4 + 8 + 16 + 32 = 63$

Pattern Explorer Beginning (pp. 20-23)

Page 20

1. $\blacktriangle = 9$ $\diamond = 13$
 $9 + 9 = 18$
 $13 + 7 = 20$

2. $\square = 8$ $\star = 11$
 $8 + 11 = 19$
 $8 + 8 + 8 = 24$

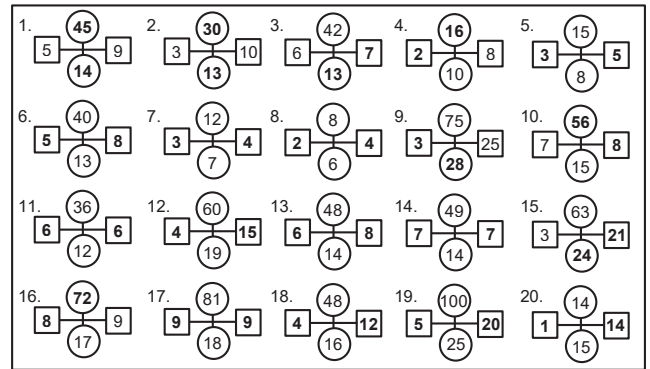
3. $\nabla = 6$ $\circlearrowleft = 12$
 $6 + 6 = 12$
 $12 + 7 = 19$

4. $\odot = 5$ $\heartsuit = 3$
 $5 + 5 + 15 = 25$
 $3 + 3 + 5 = 11$

5. $\blacklozenge = 2$ $\star = 7$ $\spadesuit = 18$
 $7 + 10 + 7 = 24$
 $2 + 7 = 9$
 $18 + 2 = 20$

6. $\smiley = 15$ $\heartsuit = 4$ $\spadesuit = 10$
 $4 + 15 + 10 = 29$
 $15 + 4 + 15 = 34$
 $15 + 13 = 28$

Page 21



Page 22

1. stage 5

2. Stage 5 has 11 toothpicks.

3. Stage 5 has 7 gumdrops.

4. Stage 6 has:
 • 13 toothpicks
 • 8 gumdrops

stage 6 stage 7 stage 8

stage	1	2	3	4	5	6	7	8
number of toothpicks	3	5	7	9	11	13	15	17
number of gumdrops	3	4	5	6	7	8	9	10

When you increase the stage number by 1, the number of toothpicks goes up by 2 and the number of gumdrops goes up by 1. Continue the pattern to find stage 12.

stage	8	9	10	11	12
number of toothpicks	17	19	21	23	25
number of gumdrops	10	11	12	13	14

1.	in	4	9	16	21	32	54
	out	17	22	29	34	45	67

2a. RULE: add 10

in	14	21	29	36	42	74
out	24	31	39	46	52	84

2b. RULE: add 22

in	6	13	18	25	42	57
out	28	35	40	47	64	79

3a.

Spencer's age	5	11	23	35	44	66
Amanda's age	13	19	31	43	52	74

Amanda is 8 years older than Spencer: when Spencer is 5, Amanda is 13, when Spencer is 11, Amanda is 19, and on.

3b. Amanda is 24. Amanda is 8 years older than Spencer: $8 + 16 = 24$.

3c. Spencer is 39. Spencer is 8 years younger than Amanda: $47 - 8 = 39$.

4a.

cost to make cake (\$)	6	9	15	22	28	36
selling price of cake (\$)	11	14	20	27	33	41

The cake's selling price is \$5 more than the cost to make the cake.

4b. The cake's selling price is \$30. The cake's selling price is \$5 more than the \$25 cost to make the cake: $\$5 + \$25 = \$30$.

4c. The cost to make the cake is \$12. The cost to make the cake is \$5 less than the \$17 selling price of cake: $\$17 - \$5 = \$12$.

5a.

Sammy's situps	20	32	45	57	70	103
Sammy's pushups	5	17	30	42	55	88

Each day Sammy does 15 less pushups than situps.

5b. Sammy does 12 pushups. Sammy does 15 less pushups than his 27 situps: $27 - 15 = 12$.

5c. Sammy does 65 situps. Sammy does 15 more situps than his 50 pushups: $50 + 15 = 65$.

Elementary Math Games (pp. 24-25)

Refer to the activity.

Crypto Mind Benders® Famous Quotations

(p. 26)

"There is no substitute for hard work".

Thomas Edison

	s	t	a	o
12	—	—	—	+
1	+	—	—	—
2	—	+	—	—
3	—	—	+	—

Answers: $s = 1$; $t = 2$; $a = 3$; $o = 12$
 If a times 6 equals o plus a plus t plus s , a must be 3 for the equation to solve. If t plus t equals s plus a , t must be 2 and s must be 1 for the equation to solve. o is then 12.

	u	e	h	w
5	—	+	—	—
11	—	—	+	—
7	—	—	—	+
4	+	—	—	—

Answers: $u = 4$; $e = 5$; $h = 11$; $w = 7$
 If w is less than 11, but greater than 6, w must be 7. If h minus w is less than h minus

u , h must be 11 and u must be 4 or 5 for the statement to be true. If h minus u is greater than h minus e , u must be 4 for the statement to be true. e is then 5.

	b	k	m	d
10	—	+	—	—
9	—	—	—	+
8	—	—	+	—
6	+	—	—	—

Answers: $b = 6$; $k = 10$; $m = 8$; $d = 9$
 If k is greater than b plus 3, k must be 10 and b must be 6 for the equation to be true. If b plus 2 equals m , m must be 8 for the equation to be true. d is then 9.

Crypto Mind Benders® Classic Jokes

(p. 27)

Why did the cat go to the computer lab every day? To play with the mouse!

	y	p	m	a
1	—	—	+	—
2	—	+	—	—
5	+	—	—	—
6	—	—	—	+

Answers: $y = 5$; $p = 2$; $m = 1$; $a = 6$
 If y does not equal 6, and y is greater than p plus m , y must be 5 for the statement to be true with the given numbers. Therefore, p and m must be either 1 or 2. Since p is greater than m , p must be 2, and m must be 1. a is then 6.

	c	e	o	g
3	—	—	+	—
4	+	—	—	—
10	—	—	—	+
12	—	+	—	—

Answers: $c = 4$; $e = 12$; $o = 3$; $g = 10$
 If g times g , plus c , equals 104, g must be 10 and c must be 4 for the equation to be true. If e minus c equals g minus 2, e must be 12 for the equation to be true. o is then 3.

	u	s	t	l
7	—	+	—	—
8	—	—	—	+
9	—	—	+	—
11	+	—	—	—

Answers: $u = 11$; $s = 7$; $t = 9$; $l = 8$
 If u minus s equals l divided by 2, l must be 8, the only number divisible by 2 that equals a whole number, u must be 11 and s must be 7, for the equation to be true. t is then 9.

Math Analogies Level 2 (p. 28)

- 5) 16 pins
8,000,000
- 6) 12,000,000
16,000,000
- 7) 13 triangles
- 8) change

Dare to Compare Level 1 (pp. 29-31)

Page 29

25. Tanya travels the longest distance.

Kevin travels the shortest distance.

Tanya travels 240 feet, Kevin travels 210 feet, and Amanda travels 225 feet.

- Tanya walks 4 feet every second for 60 seconds, so her distance is $4 \times 60 = 240$ feet.
- Kevin cycles 30 feet every second for 7 seconds, so his distance is $30 \times 7 = 210$ feet.
- Amanda runs 9 feet every second for 25 seconds, so her distance is $9 \times 25 = 225$ feet.

26. Joanne's fence costs more.

Joanne's fence costs \$350 and Remy's fence costs \$340. The perimeter of Joanne's fence is $10 + 15 + 10 + 15 = 50$ meters. Since each meter of fence costs \$7, 50 meters of fencing costs $50 \times \$7 = \350 .

The perimeter of Remy's fence is $20 + 14 + 20 + 14 = 68$ meters. Since each meter of fence costs \$5, 68 meters of fencing costs $68 \times \$5 = \340 .

27. Rachel's heart has the most beats in 1 minute. Brett's heart has the fewest beats in 1 minute.

Adam's heart beats 75 times in 1 minute, Rachel's heart beats 80 times in 1 minute, and Brett's heart beats 72 times in 1 minute.

- There are 60 seconds in 1 minute. In 20 seconds, Adam's heart beats 25 times. In $3 \times 20 = 60$ seconds, Adam's heart beats $3 \times 25 = 75$ times.
- In 120 seconds Rachel's heart beats 160 times. In $120 \div 2 = 60$ seconds, Rachel's heart beats $160 \div 2 = 80$ times.

Page 30

37. (a) 450 minutes (b) 20 minutes (c) 313 hours (d) 4,000 seconds

(a) 1 hour = 60 minutes, so 7 hours = $7 \times 60 = 420$ minutes, which is less than 450 minutes.

(b) 1 minute = 60 seconds, so 20 minutes = $20 \times 60 = 1,200$ seconds, which is more than 1,000 seconds.

(c) $31\frac{1}{3}$ hours = $3 \times 60 + 60 \div 3 = 180 + 20 = 200$ minutes, which is more than 190 minutes.

(d) 1 hour = 60 minutes, which equals $60 \times 60 = 3,600$ seconds, which is less than 4,000 seconds.

38. A pencil costs 20¢, a pen costs 35¢, and an eraser costs 60¢.

- Since 3 pencils cost 60¢, 1 pencil costs $60 \div 3 = 20$ ¢.
- 2 pencils + 1 pen cost 75¢. Since 1 pencil costs 20¢, 2 pencils cost 40¢. So $40 \div 2 = 20$ ¢. This means 1 pen costs 35¢.
- 1 pencil + 1 pen + 1 eraser cost \$1.15. So $20 \div 2 = 10$ ¢. So $10 \div 2 = 5$ ¢. So $5 \div 2 = 2.5$ ¢. This means 1 eraser costs $115 \div 2.5 = 46$ ¢.

39. Shaded region D has the largest area.

Shaded region C has the smallest area. The shaded region has an area of 30 m² for A, 28 m² for B, 27 m² for C, 36 m² for D, and 32 m² for E.

The area of a rectangle equals its length times its width.

- For A, the shaded area = $6 \text{ m} \times 5 \text{ m} = 30 \text{ m}^2$.
- For B, the rectangle area = $8 \text{ m} \times 7 \text{ m} = 56 \text{ m}^2$. The shaded area is half the rectangle area, so the shaded area = $56 \div 2 = 28 \text{ m}^2$.
- For C, the rectangle area = $6 \text{ m} \times 9 \text{ m} = 54 \text{ m}^2$. The shaded area is half the rectangle area, so the shaded area = $54 \div 2 = 27 \text{ m}^2$.
- For D, the square area = $12 \text{ m} \times 12 \text{ m} = 144 \text{ m}^2$. The shaded area is one quarter of the square area, so the shaded area = $144 \div 4 = 36 \text{ m}^2$.
- For E, the square area = $8 \text{ m} \times 8 \text{ m} = 64 \text{ m}^2$. The shaded area is two quarters, or one half, of the square area, so the shaded area = $64 \div 2 = 32 \text{ m}^2$.

103. Mason has more gas left after the trip.

Flora's car has 6 gallons left and Mason's car has 7 gallons left. Flora uses 1 gallon of gas to drive 30 miles, so she uses 5 gallons to drive 150 miles. Flora starts with 11 gallons of gas, and so ends up with $11 - 5 = 6$ gallons.

Mason uses 1 gallon of gas to drive 25 miles, so he uses $175 \div 25 = 7$ gallons to drive 175 miles. Mason starts with 14 gallons of gas, and so ends up with $14 - 7 = 7$ gallons.

104. Jonah's meal has more calories.

Jonah's meal has 700 calories and Camilla's meal has 690 calories.

Jonah's meal:

- 4 oz of chicken has 200 calories, so 8 oz of chicken has 400 calories.
- 6 oz of rice has 240 calories, so 3 oz of rice has 120 calories.
- 2 oz of beans has 60 calories, so 6 oz of beans has 180 calories.

Total calories = $400 + 120 + 180 = 700$

Camilla's meal:

- 4 oz of chicken has 200 calories, so 2 oz of chicken has 100 calories. This means 6 oz of chicken = 4 oz + 2 oz of chicken has $200 + 100 = 300$ calories.
- 6 oz of rice has 240 calories.
- 2 oz of beans has 60 calories, so 1 oz of beans has 30 calories. This means 5 oz of beans = 2 oz + 2 oz + 1 oz = $60 + 60 + 30 = 150$ calories.

Total calories = $300 + 240 + 150 = 690$

105. Robin's jar holds more water when full.

Nora's full jar holds 18 cups and Robin's full jar holds 20 cups. $1/3$ of Nora's jar is empty, and that equals 6 cups. So $2/3$ of Nora's jar equals $2 \times 6 = 12$ cups, and $3/3$ of Nora's jar (the full jar) equals $3 \times 6 = 18$ cups.

$1/4$ of Robin's jar is empty, and that equals 5 cups. So $2/4$ of Robin's jar equals $2 \times 5 = 10$ cups, $3/4$ of Robin's jar equals $3 \times 5 = 15$ cups, and $4/4$ of Robin's jar (the full jar) = $4 \times 5 = 20$ cups.

CrossNumber Math Puzzles: Sums C1

(p. 32)

13.



9	1		9	4		6	4	3	1	2	
2	5	3	4	1		3	2	1	4	5	
4	2	1		2	3	4	1		8	9	
1	3	2		3	8		3	4			
		9	7		9	7		8	7	9	
9	7		8	9		6	9		9	5	
8	6	7		2	1		8	3			
			9	4		2	9		6	3	1
2	4		1	5	4	2		2	8	4	
3	1	2	5	4		8	6	9	5	7	
1	2	4	3	6		7	9		1	2	





Math Word Problems Book 1 (p. 33)









95. a. 2 dozen b. 3 c. 27 d. 8 e. 3 f. 7

96. 16 inches

Balance Benders™ Level 1 (p. 34)

a. From 1st balance, substitute  for  on 2nd balance. (Tip 7) Rearrange. (Tip 2)

d. Remove  from both pans in answer a, so   = . (Tip 4)

e. Since  =  on 1st balance, adding them to each pan on 2nd balances is     =  . (Tip 3)

Balance Math™ & More! Level 2 (pp. 35-37)

Problem 1:

45	35	85
95	55	15
25	75	65
Total: 165		

Problem 2:

111	134	124
136	123	110
122	112	135
Total: 369		

Problem 3:

60	80	160
200	100	0
40	120	140
Total: 300		

Problem 4:

51	9	39
21	33	45
27	57	15
Total: 99		

Problem 1: $b = 80$
 $c = 30$
 $d = 50$
 $a - d = 60$

Problem 2: $a = 750$
 $c = 160$
 $d = 380$
 $b - d = 430$

Problem 3: $a = 93$
 $b = 81$
 $d = 36$
 $a - c = 66$

Problem 4: $a = 167$
 $b = 214$
 $c = 81$
 $b - d = 113$

Problem 1: $? = \frac{1}{2}$

Explanation: Divide both sides on 1st balance in fourth so $\diamond = 13$. Substitute 13 for \diamond on 2nd balance so $13 + 13 + 13 + \blacksquare \blacksquare = 40$. Remove 39 from both sides so $\blacksquare \blacksquare = 1$. Divide both sides in half so $\blacksquare = \frac{1}{2}$.

Problem 2: $? = 1\frac{1}{4}$

Explanation: Divide both sides on 2nd balance in thirds so $\triangle = \bullet$. Divide both sides on 1st balance in fourths so $\triangle = 2\frac{1}{2}$. Substitute $2\frac{1}{2}$ for \triangle above so $2\frac{1}{2} = \bullet$. Divide both sides in half so $\bullet = 1\frac{1}{4}$.

Balance Math™ Teaches Algebra (p. 38)

- From the 1st scale we know: $x = 50$.
 Substitute 50 for x in: $2x = ?$ (2nd scale). So $2(50) = 100 = ?$
- From the 1st scale we know: $4x = 20$. Divide both sides of this equation by 4 so $x = 5$.
 Substitute 5 for x in: $3x = ?$ (2nd scale). So $3(5) = 15 = ?$
- From the 1st scale we know: $\frac{1}{2}x + 2 = 10$. Subtract 2 from both sides of this equation so $\frac{1}{2}x = 8$.
 Substitute 8 for each $\frac{1}{2}x$ in: $x = ?$ (2nd scale). (Note: $1x$ is the same as x .) So $8 + 8 = 16 = ?$

Mathematical Reasoning™ Level F

(p. 39-41)

Page 39

Measure of Time	Joan	Lupe
Years Old	10 years	9 years
Days Old (Years x 365)	3,650 days	3,285 days
Hours Old (Days x 24)	87,600 hours	78,840 hours
Minutes Old (Hours x 60)	5,256,000 min.	4,730,400 min.

Page 40



Page 41

Unknown. The ad states it is the best mouthwash, but what does that mean? Is it the best because it is the least expensive, or because it has some natural ingredients, or because it is the best tasting, or because it works the best? We really don't know what the best means.

Unknown. We know that Zimmerman Mouthwash has been recommended by doctors, but that doesn't mean that it is still recommended by doctors.

False: The ad states that if your breath doesn't smell Zimmerman Fresh it will refund your money, but the ad doesn't say anything about giving your money back if the mouthwash doesn't work.

Unknown. We know that Zimmerman Mouthwash will fight germs that cause bad breath, but we don't know if it will kill the germs. It could fight the germs, but lose to the germs.

False. The ad only states that Zimmerman Mouthwash has natural ingredients. It must have some, but it could only be a few natural ingredients or mostly natural ingredients or all natural ingredients.

Math Detective® A1 (pp. 42-43)

1. 1, 2, 4, 5, 10, and 20. Sentence 12.
2. Mrs. Winterbloom, Mrs. Applecrumb, and Ms. Twinkle. Their seat numbers are 5, 10, and 20, which are factors of 20.
3. No. Mr. Lyons is in seat 19, and 19 is not a factor of 20. (19 goes into 20 once with remainder 1.)
4. a. Sentence 15.
5. Mr. Papas and Ms. Kamen. (Mr. Papas is in seat 6 and Ms. Kamen is in seat 27. 6 and 27 are multiples of 3. 3 is a factor of both 6 and 27.)
6. One is a factor of every number because 1 goes into any number OR Every number is divisible by 1.
7. d. (Everyone on the train should get a ride because every number is a multiple of 1, or 1 is a factor of every number.)

Math Word Problems Book 2 (p. 44)

- a. 16¢
- b. 84¢
- c. 13¢
- d. \$3.36
- e. 25%
- f. \$2.52
- g. 75%

Algebra Magic Tricks Book 1 (p. 45)

Refer to the activity.

Algebra Magic Tricks Book 2 (p. 46)

Refer to the activity.